

AD-772 480

FRAGMENT WEIGHT DISTRIBUTIONS FROM
NATURALLY FRAGMENTING CYLINDERS
LOADED WITH VARIOUS EXPLOSIVES

H. M. Sternberg

Naval Ordnance Laboratory
White Oak, Maryland

12 October 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified

| | | | |
|--|--|---|-----------------|
| 1. ORIGINATING ACTIVITY (Corporate author) | | 2a. REPORT SECURITY CLASSIFICATION | |
| Naval Ordnance Laboratory Silver Spring, Maryland 20910 | | Unclassified | |
| 3. REPORT TITLE | | 2b. GROUP | |
| Fragment Weight Distributions from Naturally Fragmenting Cylinders Loaded with Various Explosives. | | | |
| 4. DESCRIPTIVE NOTES (Type of report and inclusive dates) | | | |
| 5. AUTHOR(S) (First name, middle initial, last name) | | | |
| H. M. Sternberg | | | |
| 6. REPORT DATE | | 7a. TOTAL NO. OF PAGES | 7b. NO. OF REFS |
| 12 October 1973 | | 58 | 3 |
| 8a. CONTRACT OR GRANT NO. | | 9a. ORIGINATOR'S REPORT NUMBER(S) | |
| b. PROJECT NO. Task NOL-989/A | | NOLTR 73-83 | |
| c. | | 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) | |
| d. | | | |
| 10. DISTRIBUTION STATEMENT | | | |
| Approved for public release; distribution unlimited. | | | |
| 11. SUPPLEMENTARY NOTES | | 12. SPONSORING MILITARY ACTIVITY | |
| | | U.S. Army Material Systems Analysis Agency, Aberdeen Proving Ground, Md. 21005 | |
| 13. ABSTRACT | | | |
| <p>A fragment weight distribution formula is constructed by using available data. The formula involves one independent parameter, conserves mass, and uses the Mott distribution over a central portion of the weight range. Values of the parameter \bar{M}, the average weight of fragments weighing more than one grain, are assigned to 24 explosive compositions, in one particular configuration. Typical values of \bar{M} are 12 for Composition B, 17 for Tritonal, 17.5 for cast TNT, and 45 for Ammonium Nitrate - TNT (80/20).</p> | | | |

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
U. S. Department of Commerce
Springfield, VA 22151

DD FORM 1473 (PAGE 1)

S/N 0101-807-6801

Unclassified
Security Classification

UNCLASSIFIED

Security Classification

| 14 KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|--------|----|--------|----|--------|----|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Fragmentation Fragment weight distribution Explosives Munitions effectiveness Lethality | | | | | | |

FRAGMENT WEIGHT DISTRIBUTIONS FROM NATURALLY
FRAGMENTING CYLINDERS LOADED WITH VARIOUS EXPLOSIVES

By:
H. M. Sternberg

ABSTRACT: A fragment weight distribution formula is constructed by using available data. The formula involves one independent parameter, conserves mass, and uses the Mott distribution over a central portion of the weight range. Values of the parameter \bar{M} , the average weight of fragments weighing more than one grain, are assigned to 24 explosive compositions, in one particular configuration. Typical values of \bar{M} are 12 for Composition B, 17 for Tritonal, 17.5 for cast TNT, and 45 for Ammonium Nitrate - TNT (80/20).

EXPLOSION DYNAMICS DIVISION
EXPLOSIONS RESEARCH DEPARTMENT
NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

NOLTR 73-83

12 October 1973

FRAGMENT WEIGHT DISTRIBUTIONS FROM NATURALLY FRAGMENTING CYLINDERS
LOADED WITH VARIOUS EXPLOSIVES

This is the initial part of a one year effort to construct a scheme for the rapid calculation of fragment directions, velocities, and weight distributions from shells, bombs, and warheads. Previous work in this area has been collected, extended, and assembled into this report. Aside from its ultimate use in the fragment prediction scheme, the work is of interest in itself, since it compares fragmentation properties of a wide range of explosive compositions.

The work is being supported by the U.S. Army Materiel Systems Analysis Agency, under Task NOL-989/A, Fragment Prediction Method.

ROBERT WILLIAMSON II
Captain, USN
Commander

(Signature)
C. J. ARONSON
By direction

CONTENTS

| | Page |
|--|------|
| I. Introduction | 1 |
| II. The Weight Distribution Formula | 3 |
| III. The Fit to the Data | 9 |
| IV. Effect of Explosive Composition | 11 |
| V. Comparison with Preformed Fragments | 12 |
| References | 13 |

ILLUSTRATIONS

| Figure | Title |
|--------|---|
| 1 | Percent by Weight of Fragments Weighing Less Than One Grain |
| 2 | Regions Used for the Fragment Weight Distribution Formula |
| 3 | The Exponent q , Used in Region III, vs \bar{M} |
| 4 | Fragment Weight Distributions for Several Values of \bar{M} |
| 5 | $\bar{N}(m)$ vs the Parameter \bar{M} for Fixed Values of m |
| 6 | The NWL Data |
| 6-1 | $m = 5$ |
| 6-2 | $m = 10$ |
| 6-3 | $m = 15$ |
| 6-4 | $m = 25$ |
| 6-5 | $m = 50$ |
| 6-6 | $m = 75$ |
| 6-7 | $m = 150$ |
| 7 | Arrangement of Charge and Initiator for Firing of NOL Charges |

CONTENTS (cont.)

| Figure | Title |
|--------|---|
| 8 | Cumulative Fragment Weight Distribution |

- 8-1 Baratol (Cast)
- 8-2 Composition B (Cast)
- 8-3 Cyclotol (Cast)
- 8-4 H-6 (Cast)
- 8-5 HBX-1 (Cast)
- 8-6 HBX-3 (Cast)
- 8-7 Minol 2 (Cast)
- 8-8 Pentolite (Cast)
- 8-9 PTX-1 (Cast)
- 8-10 PTX-2 (Cast)
- 8-11 Torpex (Cast)
- 8-12 Tritonal (Cast)
- 8-13 TNT (Cast)
- 8-14 Ammonium Nitrate - TNT (Pressed)
- 8-15 BTNEN/Wax (Pressed)
- 8-16 Composition A3 (Pressed)
- 8-17 Explosive D (Pressed)
- 8-18 MOX-2B (Pressed)
- 8-19 Nitroguanidine (Pressed)
- 8-20 Pentolite (Pressed)
- 8-21 RDX/Wax (97/3) (Pressed)
- 8-22 RDX/Wax (95/5) (Pressed)
- 8-23 RDX/Wax (88/12) (Pressed)
- 8-24 RDX/Wax (85/15) (Pressed)
- 8-25 Tetryl (Pressed)
- 8-26 TNT (Pressed)

| | |
|---|--|
| 9 | Cumulative Fragments per Lb vs Design Weight |
|---|--|

TABLES

| Table | Title |
|-------|--|
| I | List - BASIC Computer Program for Fragment Weight Distribution Calculation |
| II | Computer Program Outline |
| III | Region Boundary and Envelope Points for Various Values of \bar{M} |
| IV | Calculated Values of Parameters p , n_{II} , μ_{II} , and N_2 |

TABLES (Cont.)

| Table | Title |
|-------|--|
| V | Calculated Values of Parameters n_{III} , μ_{III} , and q |
| VI | Calculated Weight Fractions in Various Regions |
| VII | Average Weight of Fragments Weighing More Than One Grain from Steel Cylinders Filled with Various Cast Explosives |
| VIII | Average Weight of Fragments Weighing More Than One Grain from Steel Cylinders Filled with Various Pressed Explosives |

I. Introduction

When a steel cylinder is filled with a high explosive and the explosive is detonated the fragment weight distribution depends on the location of the initiation point, or points, the cylinder diameter, the wall thickness, the type and treatment of the steel, and the composition and density of the explosive. A scheme to get the fragment weight distribution, analytically, once these variables are specified, has been a goal of warhead research for over thirty years. Two aspects of the problem are considered here, namely, the construction of a one parameter family of fragment weight distributions, and the assignment of values of this parameter to various high explosives, for one particular steel type, cylinder size, and initiation point location.

Two sets of fragment weight distribution data were used. The first set (the NWL data) is from work at the Naval Weapons Laboratory, Dahlgren, Virginia¹, on the effect of various steels and treatments on fragment size. It consists of data sheets covering 116 sawdust pit firings of Composition B loaded, uncapped, steel cylinders ranging in diameter from 40 mm to 8 in., with explosive/metal mass ratios (C/M) between 0.1 and 0.4. The second set of data (the NOL data) is mainly from a 1953 NOL report by Solem, Shapiro, and Singleton², where the effect of explosive composition on fragmentation was studied.

The examination of the NWL¹ and NOL² data showed that the commonly used Mott distribution³ gives a reasonably good fit only in a central portion of the fragment weight range. There are more small fragments and fewer large fragments than the Mott formula predicts. This difficulty is overcome here with a more complete formula which involves only one independent parameter, conserves mass, and uses the Mott distribution where it applies. The measured weight distribution is somewhat dependent on the fragment recovery technique. In the NWL and NOL experiments considered here the ejected fragments were collected in sawdust, after crossing an air gap. This was followed by magnetic separation.

The analysis here makes use of the envelope of the fragment weight distribution curves. The envelope will be seen to have a central part

NOLTR 73-83

in the construction of the weight distribution formula, in finding the optimum value of the independent parameter for particular applications, and in comparing natural fragmentation with preformed or controlled fragments.

II. The Weight Distribution Formula

The following notation will be used. Let m be the fragment weight, in grains unless otherwise specified (7000 grains = 1 lb). For a particular metal casing, let \tilde{W}_T be the total casing weight in grains, and let $\tilde{W}(m)$ and $\tilde{N}(m)$ be, respectively, the weight in grains and the cumulative number of fragments of weight greater than m . Let

$$\left. \begin{aligned} W &= \tilde{W}/\tilde{W}_T \\ N &= \tilde{N}/\tilde{W}_T \quad (\text{fragments/grain}), \end{aligned} \right\} \quad (1)$$

and

$$\bar{N} = 7000 N \quad (\text{fragments/lb.})$$

The symbols N and $N(m)$ have the same meaning. Similarly, $\tilde{N} = \tilde{N}(m)$, $\tilde{W} = \tilde{W}(m)$, $W = W(m)$, $\bar{N} = \bar{N}(m)$.

Note that to conserve mass

$$dW = m dN. \quad (2)$$

From this

$$\int_0^{\tilde{N}_0} m d\tilde{N} = \tilde{W}_T,$$

where $N_0 = N(0)$, the total number of fragments. In normalized form

$$\int_0^{N_0} m dN = 1, \text{ or } \int_0^{\bar{N}_0} m d\bar{N} = 1 \quad (3)$$

Here N_0 is the total number of fragments per grain, and \bar{N}_0 is the total number of fragments per lb.

The Mott Formula for the fragment weight distribution can be written

$$N(m) = N_0 \exp \left[-(m/\mu)^{1/2} \right], \quad (4)$$

where N_0 and μ are constants for a particular weight distribution. The value of the constant N_0 in (4) is consistent with (3), i.e., $N_0 = N(0)$, the total number of fragments per grain. The graph of (4) is a straight line in the $m^{1/2}$, $\log N$ plane, where $-\mu^{1/2} \log e$ is the slope and $\log N_0$ is the ordinate intercept.

If the Mott formula is assumed to hold for the entire weight distribution then the constants N_0 and μ are related, through the conservation of mass, by

$$N_0 = 1/(2\mu). \quad (5)$$

To show this, note from (4) that $m = \mu[\ln(N_0/N)]^2$. Inserting this into (3) leads to (5). The Mott distribution is thus seen to depend on only one independent parameter.

The cylinder fragmentation data examined here shows that in most cases the Mott distribution (4) holds only over a central portion of the weight range. There are more small fragments and fewer large fragments than the Mott formula predicts. To account for this type of distribution a three part formula was constructed. Account is taken of the total weight of fragments weighing less than one grain, but the number and weight distribution of these fragments are not considered. The fragments weighing more than one grain are divided into three groups. The Mott formula (4) is used for the central portion of the fragment weight range, the Mott formula with the exponent 1/2 replaced by a value between 0.5 and 1 is used for the larger fragments, and a power law is used for the smaller fragments. The weight distribution is written

$$\text{I : } N = N_1 m^p, \quad m_1 < m \leq m_2, \quad m_1 = 1 \quad (6)$$

$$\text{II : } N = n_{\text{II}} \exp [-(m/\mu_{\text{II}})^{1/2}], \quad m_2 < m \leq m_3 \quad (7)$$

$$\text{III : } N = n_{\text{III}} \exp [-(m/\mu_{\text{III}})^q], \quad m > m_3 \quad (8)$$

The subscripts I, II, and III refer to regions and the subscripts 1, 2, and 3 refer to the region boundaries. As in the Mott formula, the distribution is assumed to be completely determined by a single parameter. The parameter used in the NWL work is used here, namely, \bar{M} , the average weight of fragments weighing more than one grain, i.e.,

$$\bar{M} = \tilde{W}(1)/\tilde{N}(1) = W(1)/N(1). \quad (9)$$

The notation $N_j = N(m_j)$, $W_j = W(m_j)$ will be used. Thus, since $m_1 = 1$, the left boundary of Region I (Equation 6),

$$\tilde{N}_1 = \tilde{N}(1), \tilde{W}_1 = \tilde{W}(1), N_1 = N(1), W_1 = W(1),$$

but N_2 refers to the cumulative number of fragments, per grain, of weight greater than m_2 grains. In this notation $\bar{M} = W_1/N_1$.

Since the weight distribution is assumed to be entirely determined by the single parameter \bar{M} , the quantities N_1 , n_{II} , n_{III} , μ_{II} , μ_{III} , p , q , m_2 , and m_3 , appearing in (6) - (8) are functions of \bar{M} .

The weight fraction of fragments weighing less than one grain was found from the experimental data in reference 1. The data were fitted to the form

$$\frac{\tilde{W}_T - \tilde{W}_1}{\tilde{W}_T} = \frac{K}{\bar{M} + K}, \quad (10)$$

with K a constant. Then from (1), (9), and (10)

$$W_1 = \bar{M}/(\bar{M}+K), \quad (11)$$

and

$$N_1 = 1/(\bar{M}+K). \quad (12)$$

Here, W_1 and N_1 are, respectively, the weight fraction and number of fragments per grain, of weight larger than one grain. Note the distinction between W_1 , the weight fraction weighing more than one grain and W_I , the weight fraction of fragments in Region I.

When the weight distributions N vs m are plotted for various values of the parameter \bar{M} , the curves are seen to have an envelope, determined by the condition

$$(\partial N / \partial \bar{M})_m = 0 \quad (\text{envelope}). \quad (13)$$

From another viewpoint, this envelope is the locus of the maxima of the N vs \bar{M} plotted for fixed values of m . Denote the envelope by $N_e(m_e)$ and note that m_e , and consequently N_e , is a function of \bar{M} .

An examination of available experimental^{1,2} data shows that the envelope points (m_e, N_e) lie in Region II, or at the II-III boundary, for all values of \bar{M} considered. We thus have, from (7) and (13), on the envelope

$$\frac{1}{n_{II}} \frac{dn_{II}}{d\bar{M}} - m_e^{1/2} \frac{d\mu_{II}}{d\bar{M}}^{-1/2} = 0 \quad (\text{envelope}). \quad (15)$$

Now, (7) holds on the envelope, so that

$$n_{II} = N_e \exp [(m_e / \mu_{II})^{1/2}]. \quad (16)$$

Differentiating (16) with respect to \bar{M} and combining the result with (15) leads to

$$\frac{d(m_e^{1/2})}{dN_e} = - \frac{\mu_{II}^{1/2}}{N_e}. \quad (17)$$

A satisfactory fit to the experimentally determined envelope had, for \bar{M} larger than 4 grains, the form

$$m_e = C \bar{M}^{B_2}, \quad N_e = L / m_e \quad (\bar{M} > 4 \text{ grains}) \quad (18)$$

where C , B_2 , and L are constants. From (17) and (18),

$$\mu_{II} = (m_e / 4) = (L / 4 N_e) = (C / 4) \bar{M}^{B_2}, \quad (19)$$

and from (16), (18), and (19),

$$n_{II} = (L/C) \bar{M}^{-B_2} \exp[2] \quad (20)$$

Now, from (7) and (20),

$$N_2 = n_{II} \exp [-(m_2/\mu_{II})^{1/2}] \quad (21)$$

and

$$N_3 = n_{II} \exp [-(m_3/\mu_{II})^{1/2}] \quad (22)$$

Also, using (6) and (7) at the I-II boundary, the exponent p in (6) is

$$p = 1/\ln (N_2/N_1)/\ln m_2 \quad (23)$$

To find n_{III} , q , and μ_{III} in (8), note first that, since (8) holds at the II-III boundary,

$$n_{III} = N_3 \exp [(m_3/\mu_{III})^q] \quad (24)$$

where m_3 is known, from the experimental data. Now stipulate that N and $(\partial N/\partial m)_{\bar{M}}$ are continuous across the II-III boundary. This leads, using (7) and (8), to

$$\mu_{III} = [2qm_3^{q-1/2} \mu_{II}^{1/2}]^{1/q} \quad (25)$$

We can now get q , the exponent in III, by conserving mass, i.e., by satisfying (2). Denote the normalized weights of the fragments in each of the regions by W_I , W_{II} , and W_{III} . From (17),

$$W_I + W_{II} + W_{III} = \frac{\bar{M}}{\bar{M}+K} \quad (26)$$

The weights W_I and W_{II} calculated with (2) are

$$W_I = \frac{N_1}{1 + (1/p)} [1 - (N_2/N_1)^{(1 + 1/p)}] \quad (27)$$

$$W_{II} = N_2 [m_2 + 2(m_2 \mu_{II})^{1/2} + 2\mu_{II}] - N_3 [m_3 + 2(m_3 \mu_{III})^{1/2} + 2\mu_{III}] . \quad (28)$$

Also, using (2),

$$W_{III} = \int_0^{N_3} \mu_{III} \left(\ln \frac{n_{III}}{N} \right)^{1/q} dN \quad (29)$$

Equation (29) was solved for q by the method of false position, with the intergral evaluated by Simpson's rule at each stage. The quantity W_{III} is available from (26)-(28), while μ_{III} , and n_{III} are known functions of q from (24) and (25).

It turned out to be feasible to take the envelope of (7) as the II-III boundary, i.e., $m_3 = m_e$, $N_3 = N_e$.

To sum up, the fragment weight distribution (6)-(8) considered as a one parameter family, can be calculated once the weight fraction less than one grain, the envelope, and the region separation lines are obtained from the experimental data. The distributions need be calculated only once for each value of \bar{M} . To the extent that the one parameter assumption is valid, the entire weight distribution of fragments weighing more than one grain is determined, for a naturally fragmenting cylinder, by specifying the value of \bar{M} .

A computer program, in BASIC, to calculate the weight distribution with the scheme outlined in this section is listed in Table I. Table II is an outline of the computer program. It also contains a list of the parameters, the corresponding notations in the text, and the equations used to calculate them. Values of the parameters for the region divisions and the particular constants L , C , B_2 , and K used here are listed, for various values of \bar{M} , in Tables III - VI. Tables III - V can be used to construct the weight distributions directly, from Equations (6)-(8), without the computer program. Table VI shows the weight fraction in each of the regions, for various values of \bar{M} .

III. The Fit to the Data

Both the NWL¹ and the NOL² data were used to make a fit to the form in Equations (6)-(8). Each data sheet furnished by NWL covered one sawdust pit firing and contained lists of the number and total weight of fragments in the weight ranges 0-1, 1-2, 2-5, 5-10, 10-25, 25-50, 50-75, 75-150, and over 150 grains. The NOL results² appear as plots of $N(m)$ vs m for 0.5 gram (7.7 grain) weight intervals starting at 0.5 grams. Each plotted point in the NOL report is an average of data from three different firings. The fit here was made entirely by plotting the data in convenient ways and drawing best lines by eye.

The weight fraction of fragments weighing less than one grain was found from the NWL data¹ alone (the smallest weight range in the NOL data is 0.5-1.0 gram). In Figure 1 the NWL data is used to plot, on reciprocal ruling paper, the percent by weight of fragments weighing less than one grain vs \bar{M} . Note, in the figure, that these fragments are 11% of the total weight when \bar{M} is 5 grains, 6% when \bar{M} is 10 grains and 2% when \bar{M} is 30 grains. The data was fitted to (10), with $K = 0.6$, so that from (12),

$$\bar{N}_1 = 7000/(\bar{M}+0.6). \quad (30)$$

The envelope (18), constructed to fit both the NWL and the NOL data, is

$$m_e = 2050/\bar{N}_e = 0.236 \bar{M}^{1.77} \quad (\bar{M} > 4 \text{ grains}). \quad (31)$$

A fit made for the boundary line between Regions I and II gave

$$m_2 = 1 + (2/9)(\bar{M}-1)^{5/4}. \quad (32)$$

Recall, from Section II, that the envelope will be taken as the boundary between Regions II and III, i.e.,

$$m_3 = m_e. \quad (33)$$

The remaining quantities in the weight distribution formula (6)-(8) can now be calculated by the method outlined in Section II. We get μ_{II} , n_{II} , N_2 , N_3 and p from (19)-(23), and q , n_{III} and μ_{III} by the numerical solution of (24), (25) and (29).

The region boundaries (32) and (33) are shown in Figure 2. The exponent q , used in Region III, is plotted vs \bar{M} in Figure 3. Figure 4 shows the weight distributions for several values of \bar{M} calculated from (6)-(8) and (30)-(33). The heavy straight lines correspond to Region II where the Mott distribution with the exponent 0.5 is used. In Figure 5 the fit to the weight distributions is plotted in a different way, as $\bar{N}(m)$ vs the parameter \bar{M} for fixed values of m . The curve for $m = 1$ is just (30).

The NWL data were plotted in the form shown in Figure 5 by taking, for each m corresponding to the end of a weight interval, one \bar{M} point from each data sheet. In Figure 6 the data is shown in this form together with the fitted curves in Figure 5. The maxima of some of the fitted curves is a bit higher than a best fit to the data would give. This came about from the imposed requirement that a single fit be made to both the NWL and the NOL data.

IV. Effect of Explosive Composition

The NOL results consist of the data for 15 different explosives treated in NAVORD Report 2933², together with unpublished data for eight additional explosives. These additional data, obtained in 1952-1953, were taken from NOL Project Notebook 4352. In all cases the explosives were loaded into AISI 1045 steel tubes, 9 inches long, 2.0 inches ID, 2.5 inches OD, with a hardness about Rockwell 100-B. The experimental arrangement is shown in Figure 7. Since the densities of the various explosives are different, the explosives are being compared on a volume, rather than weight, basis. The cylinder breakup will vary with the method of initiation, the wall thickness, the cylinder diameter, and the type of steel, as well as with the explosive composition. Hence, the values of \bar{M} obtained for the different explosives apply only to this system.

To assign a value of \bar{M} to each explosive, the weight distributions were first calculated with (6)-(8) for values of \bar{M} ranging from 4 to 50 grains at intervals of 0.5 grains. Then, for each explosive, the weight distributions corresponding to the different values of \bar{M} were superimposed on the data points and the value of \bar{M} which fit the data best was selected.

The NOL data for the various explosives, together with the calculated weight distribution corresponding to the assigned \bar{M} values, are shown in Figure 8.

Results for all the explosive compositions studied are summarized in Tables VII and VIII. Note here that the smallest value of \bar{M} , 10.5, was obtained with (90/10) BTNEN/Wax. Among the explosives practical for military use Composition B and Composition A3, both with $\bar{M} = 12$, are at the low end of the range. The fragmentation from these is significantly finer than that from cast TNT ($\bar{M} = 17.5$) or pressed TNT ($\bar{M} = 22$). Fragmentation from some of the aluminized compositions is in between that from Composition B and TNT. For example, \bar{M} is 13.5 for HBX-1, 15.5 for H-6, and 17 for Tritonal. Note that 80/20 Ammonium Nitrate/TNT produced the largest average fragment weight, with $\bar{M} = 45$.

V. Comparison with Preformed Fragments

The concept of a design weight m_d is useful in connection with applications. One wishes to maximize $N(m_d)$, the number of fragments weighing more than m_d grains. For each design weight m_d there will be a value of \bar{M} , corresponding to a weight distribution, for which $\bar{N}(m_d)$ is a maximum (set $m = m_d$ in Figure 6). The locus of these maxima for different values of m_d is the envelope (13), also shown in Figure 6.

The relation between preformed and natural fragmentation for a given design weight is shown in Figure 9. This contains a $\log \bar{N}$ vs $\log m$ plot of the weight distributions from natural fragmentation for different values of \bar{M} , calculated with (6)-(8). The envelope, for \bar{M} bigger than 4 grains, is a straight line parallel to $7000/m_d$, the number of fragments available from preformed fragmentation. A comparison of the envelope and the preformed fragment line shows that the upper bound for $\bar{N}(m_d)$ from natural fragmentation is 29% ($100 \times 2050/7000$ - see Sections II and III) of $\bar{N}(m_d)$ from uniform fragments of weight m_d . This upper bound for natural fragmentation is solely a property of the way the steel casings break up. In any particular instance, it may not be attainable, because of cost, strength, or material availability problems.

REFERENCES

1. Private Communication, from D. S. Malyevac, Naval Weapons Laboratory, Dahlgren, Virginia, 17 November 1970.
2. A. D. Solem, M. Shapiro, and B. N. Singleton, Jr., "Explosives Comparison for Fragmentation Effectiveness," U.S. Naval Ordnance Laboratory, White Oak, Maryland, NAVORD Report 2933, 11 Aug 1953.
3. See R. W. Gurney and J. N. Sarmousakis, "The Mass Distribution of Fragments from Bombs, Shells, and Grenades," Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland, BRL Report No. 448, 7 Feb 1944.

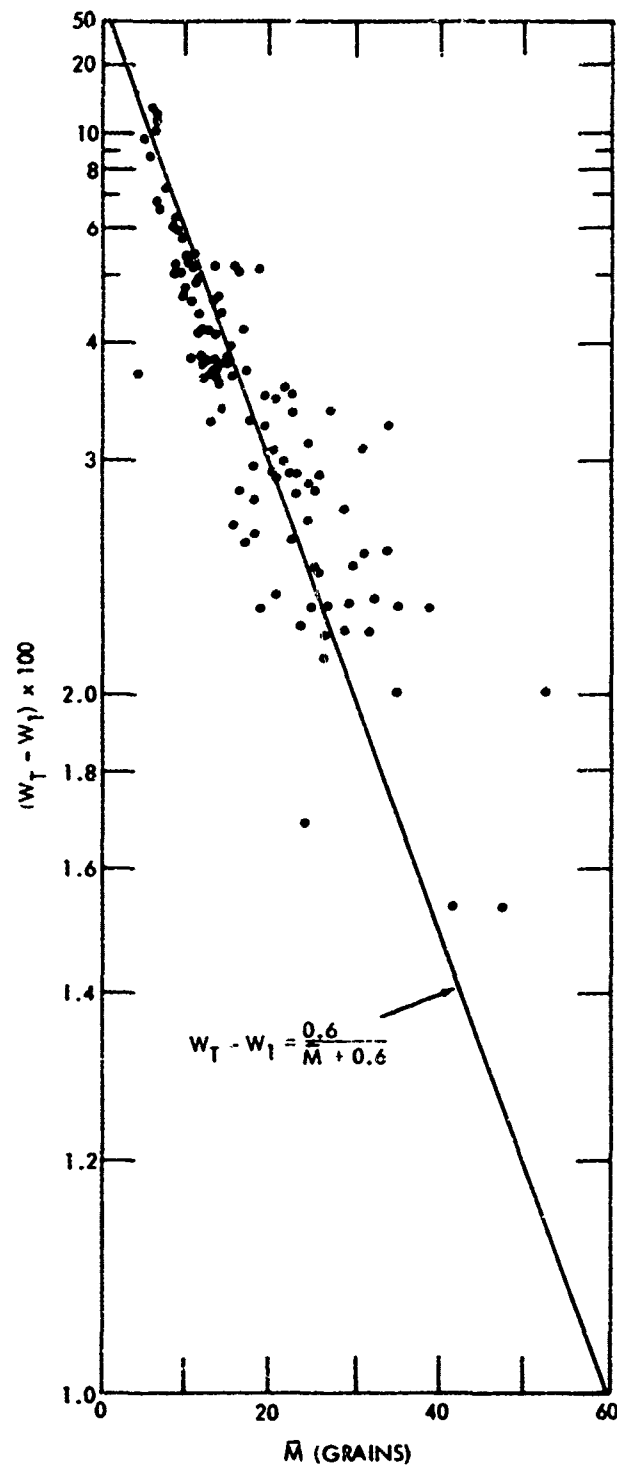


FIG. 1 PERCENT BY WEIGHT OF FRAGMENTS WEIGHING LESS THAN ONE GRAIN.

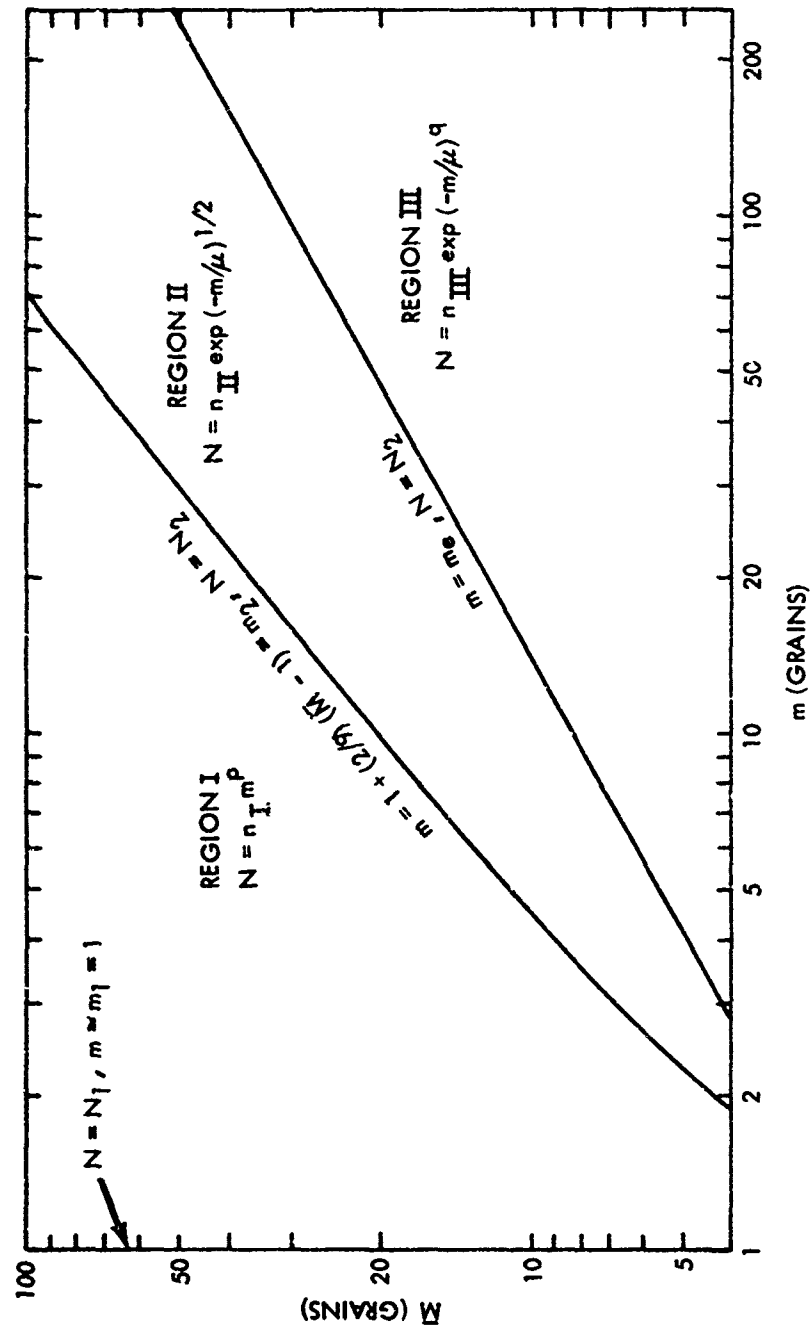


FIG. 2 REGIONS USED FOR THE FRAGMENT WEIGHT DISTRIBUTION FORMULA

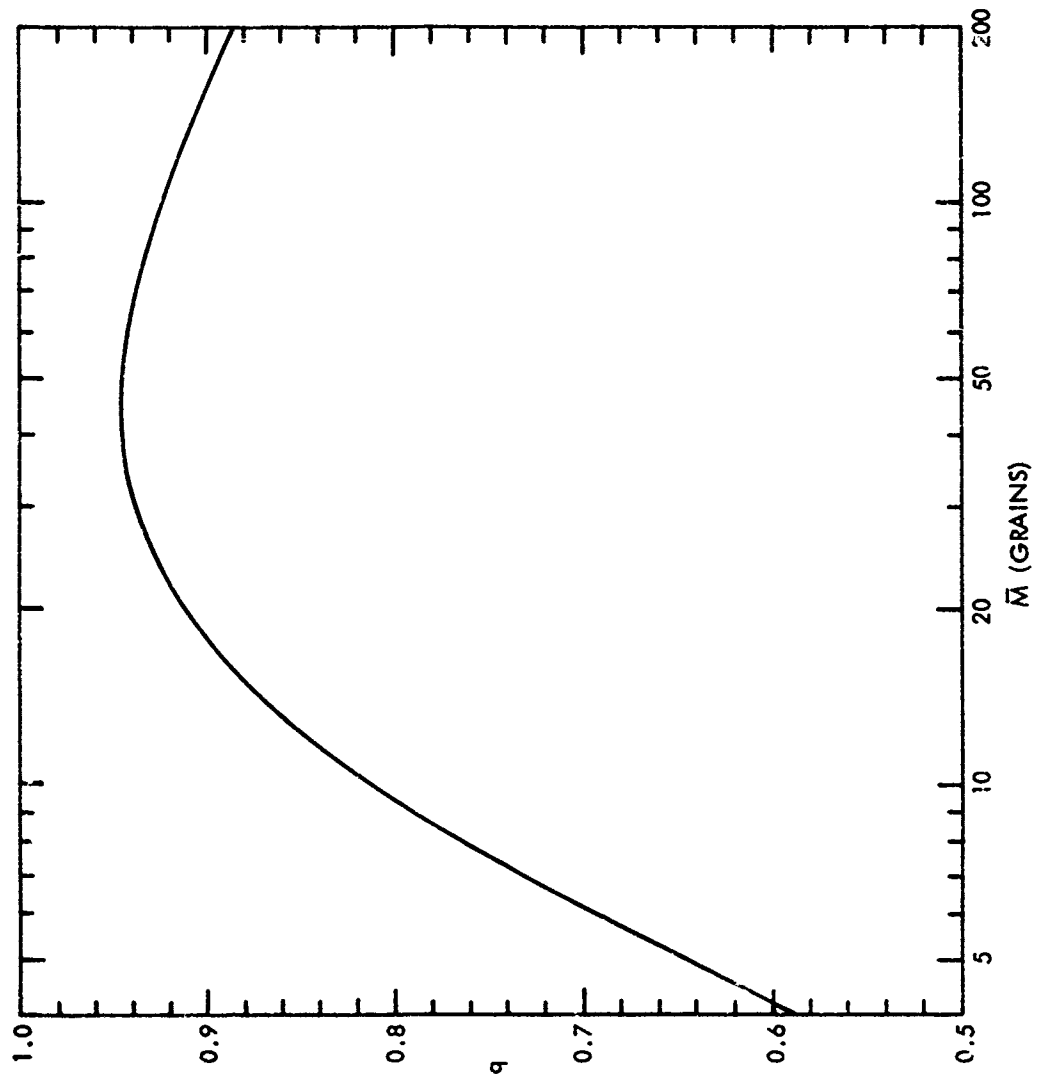


FIG. 3 THE EXPONENT q , USED IN REGION III, VS \bar{M}

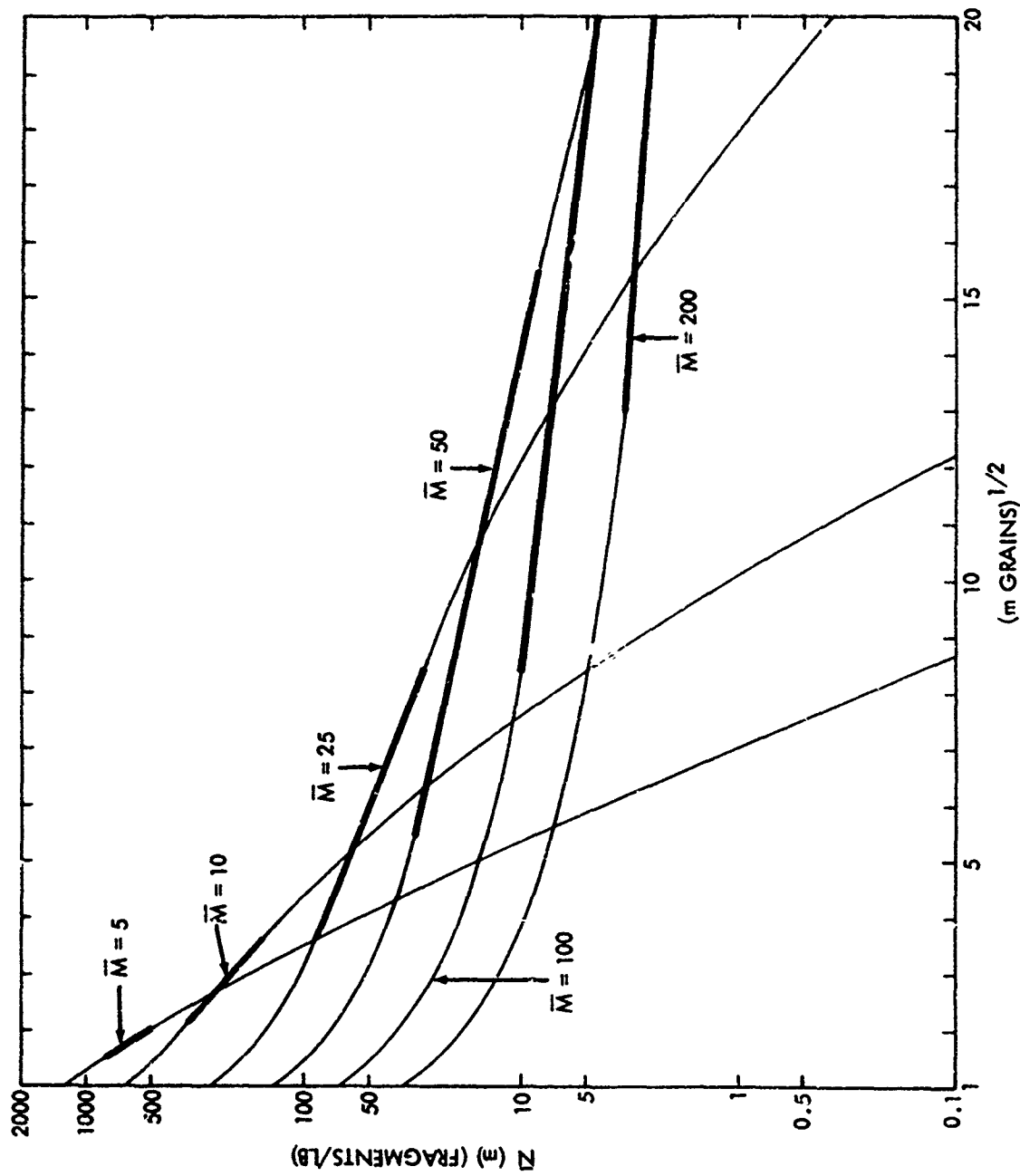


FIG. 4 FRAGMENT WEIGHT DISTRIBUTIONS FOR SEVERAL VALUES OF \bar{M} .
CALCULATED WITH EQUATIONS (6) - (8).

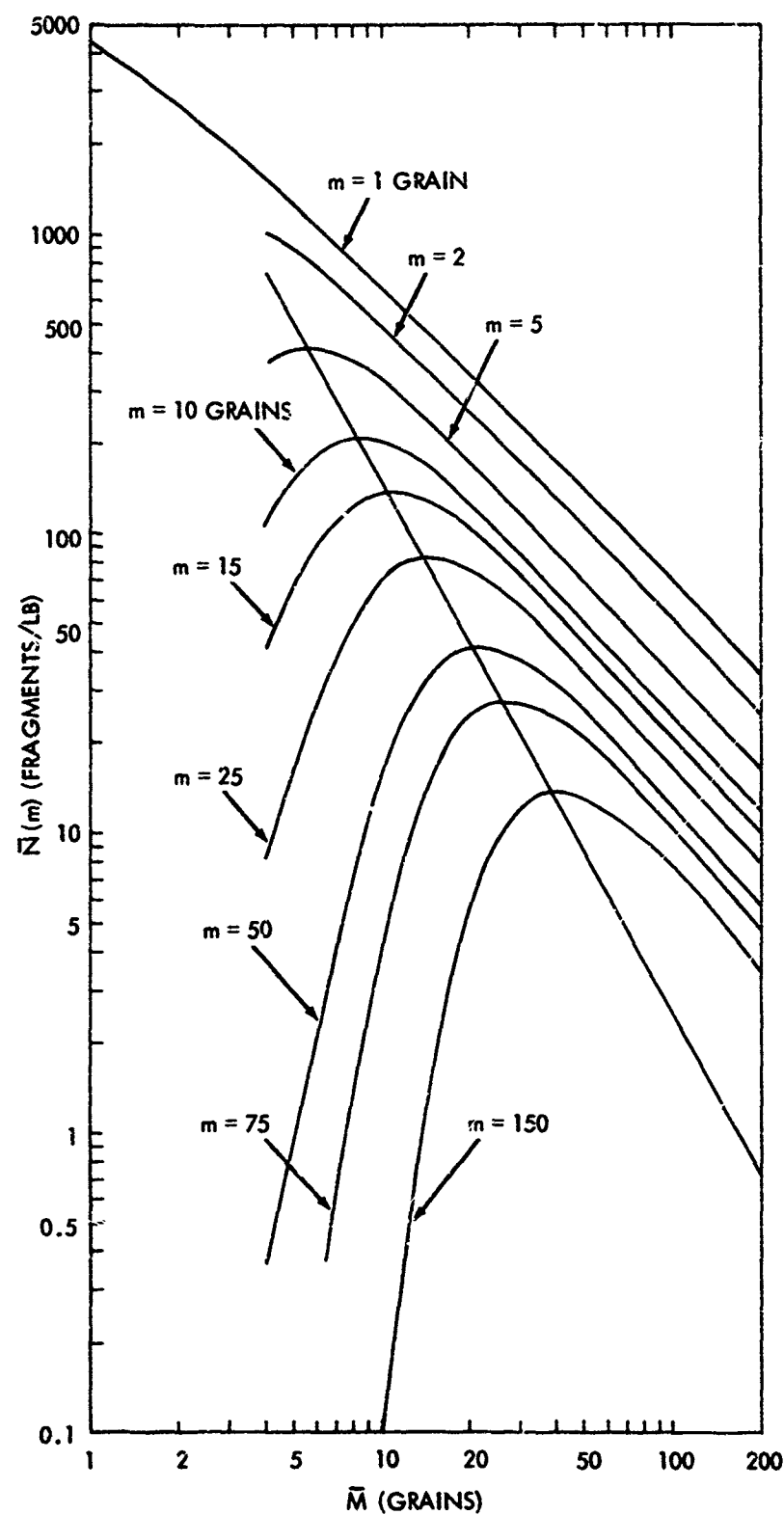


FIG. 5 $\bar{N}(m)$ VS THE PARAMETER \bar{M} FOR FIXED VALUES OF m . CALCULATED WITH EQUATIONS (6) - (8).

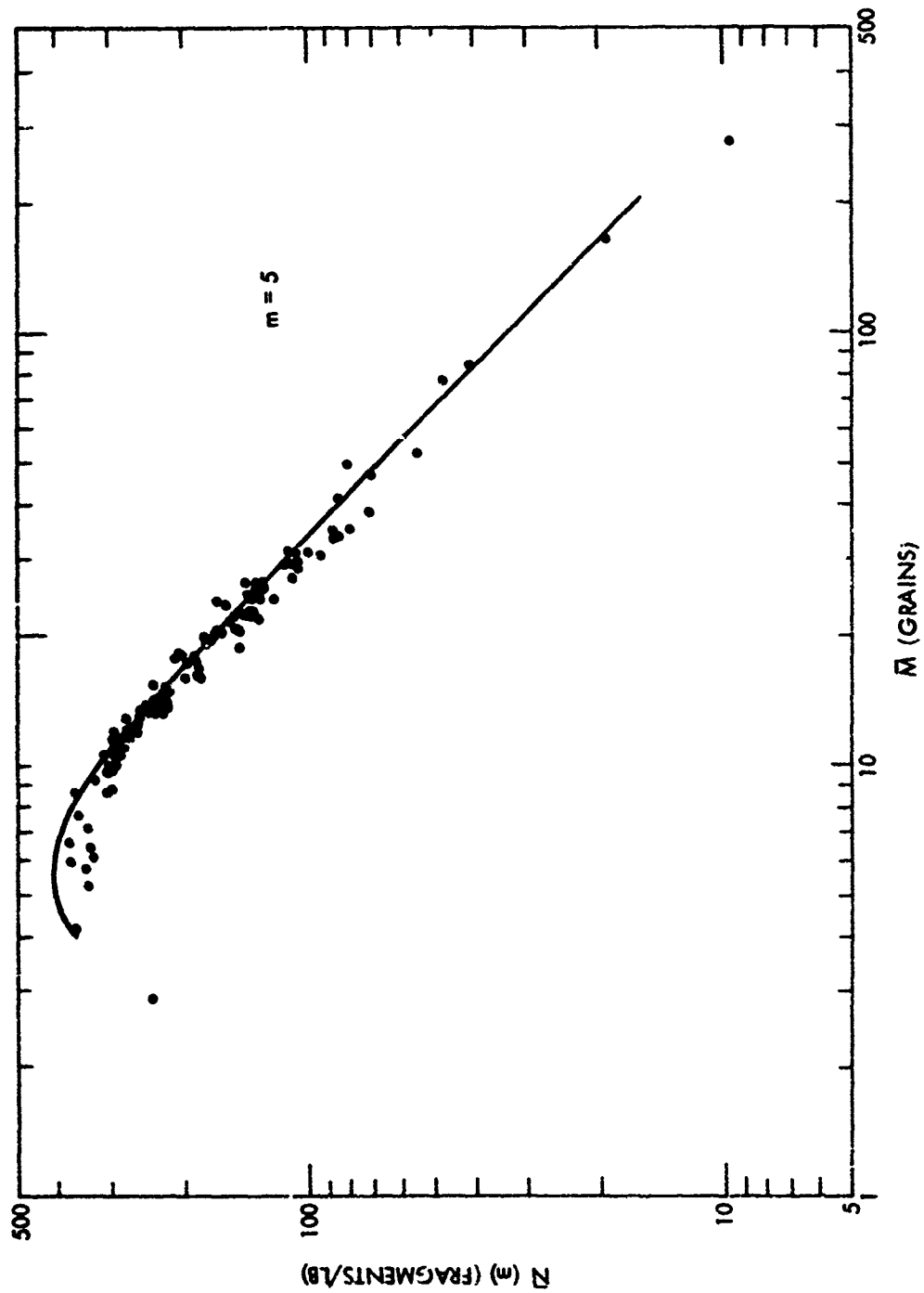


FIG. 6-1 THE NWL DATA (SUPPLEMENT TO REF 1). THE FITTED CURVES WERE CALCULATED WITH EQUATIONS (6) - (8).

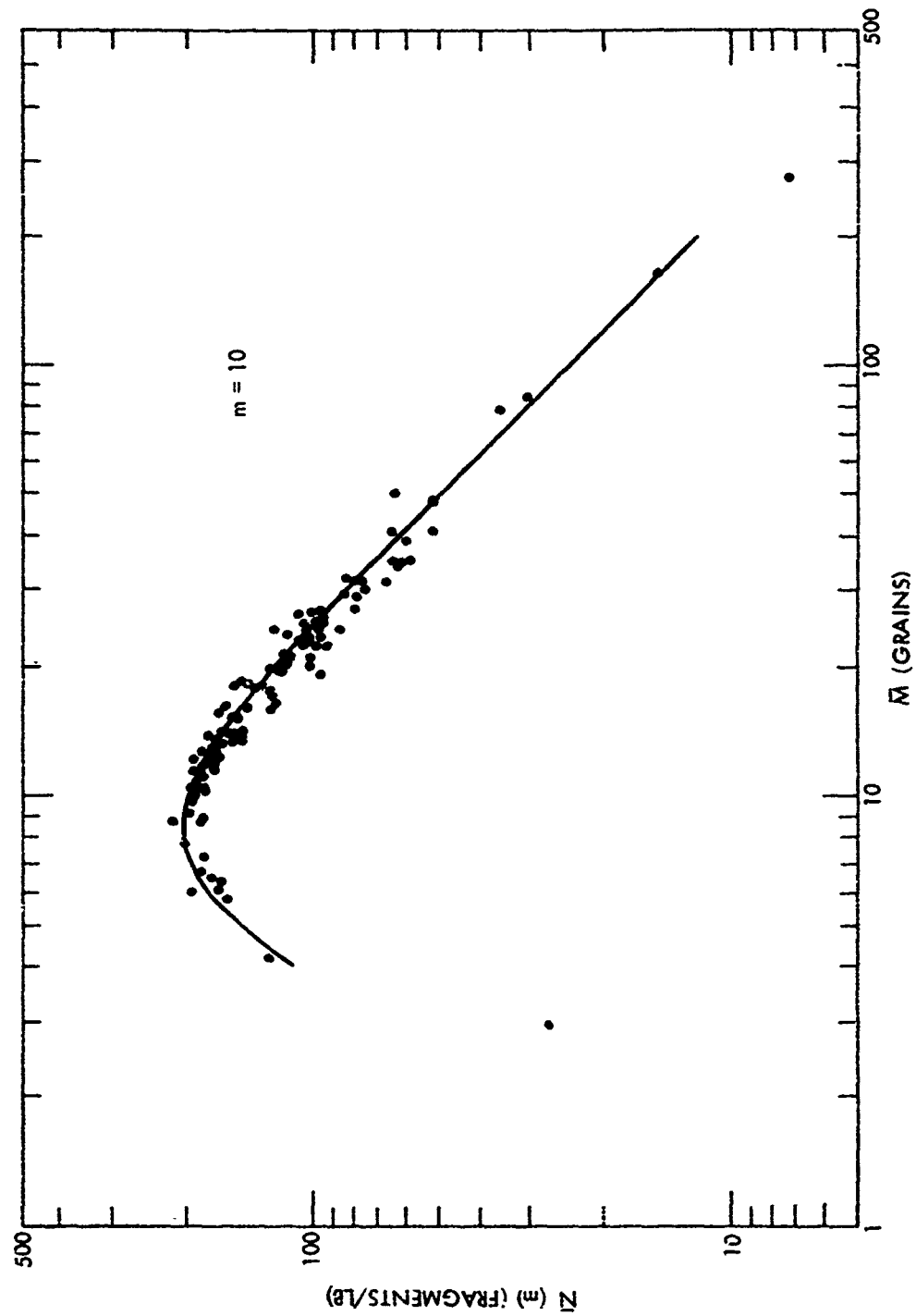


FIG. 6-2 THE NWL DATA (SUPPLEMENT TO REF 1). THE FITTED CURVES WERE CALCULATED WITH EQUATIONS (6) - (8). (CONT)

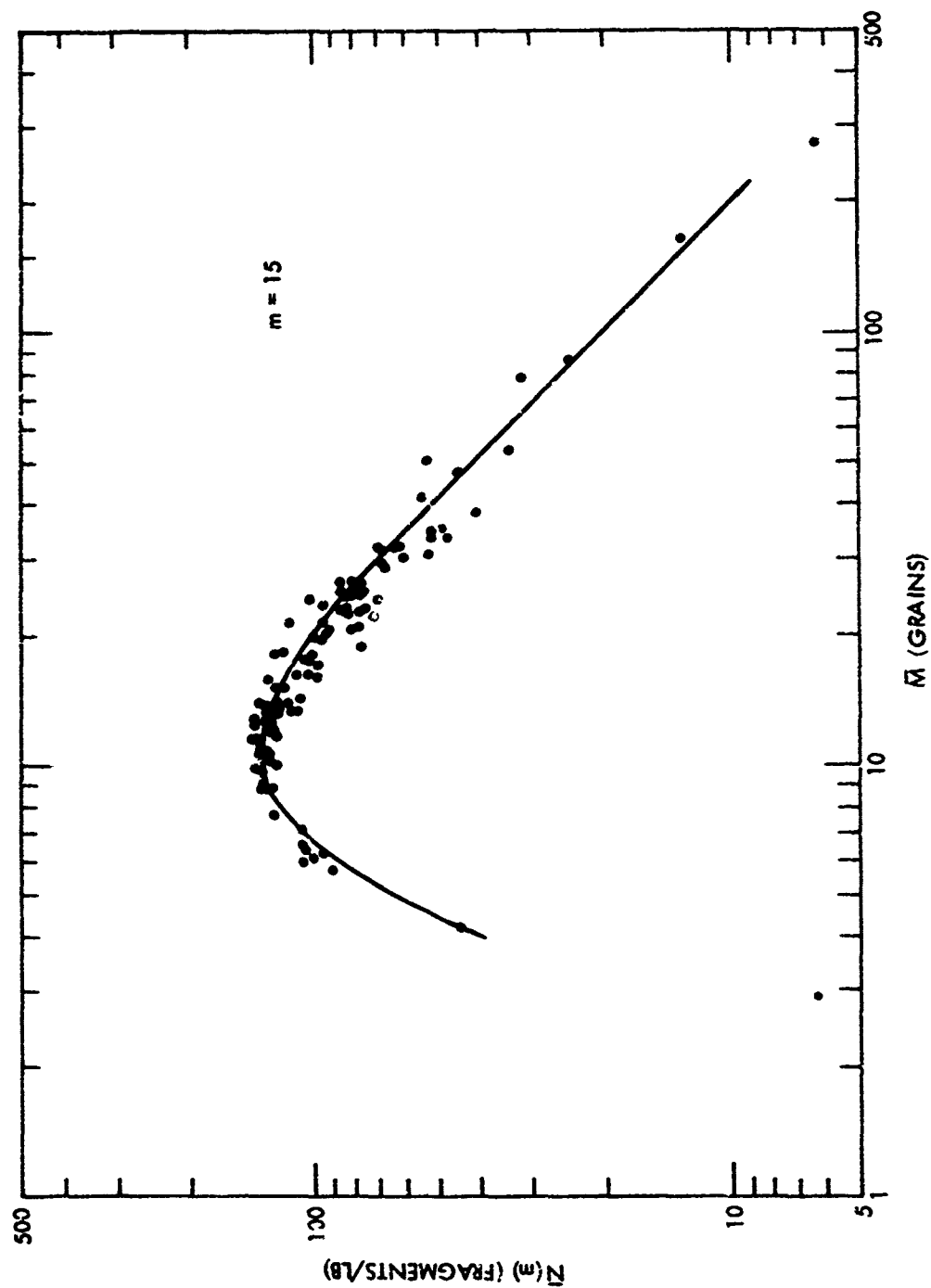


FIG. 6-3 THE NWL DATA (SUPPLEMENT TO REF 1). THE FITTED CURVES WERE CALCULATED WITH EQUATIONS (6) - (8). (CONT)

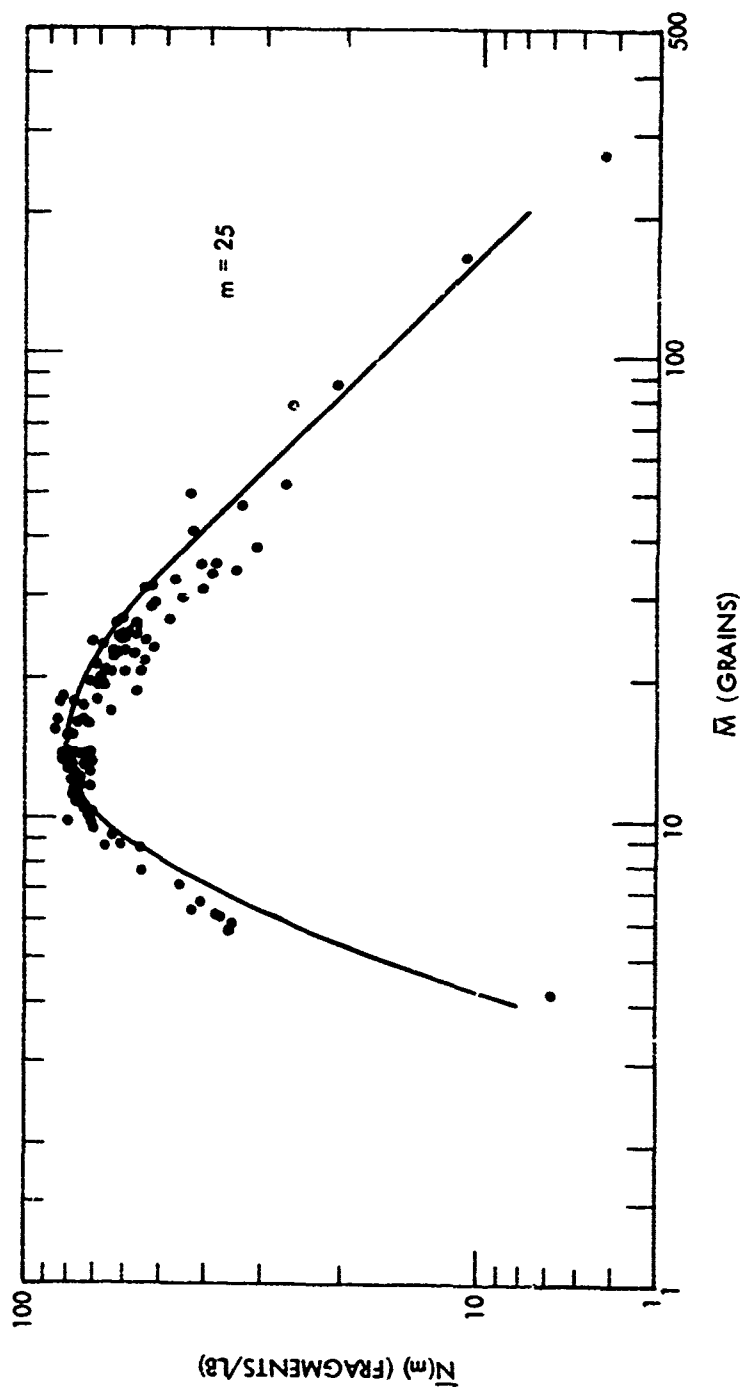


FIG. 6-4 THE NWL DATA (SUPPLEMENT TO REF 1). THE FITTED CURVES WERE CALCULATED WITH EQUATIONS (6) - (8). (CONT)

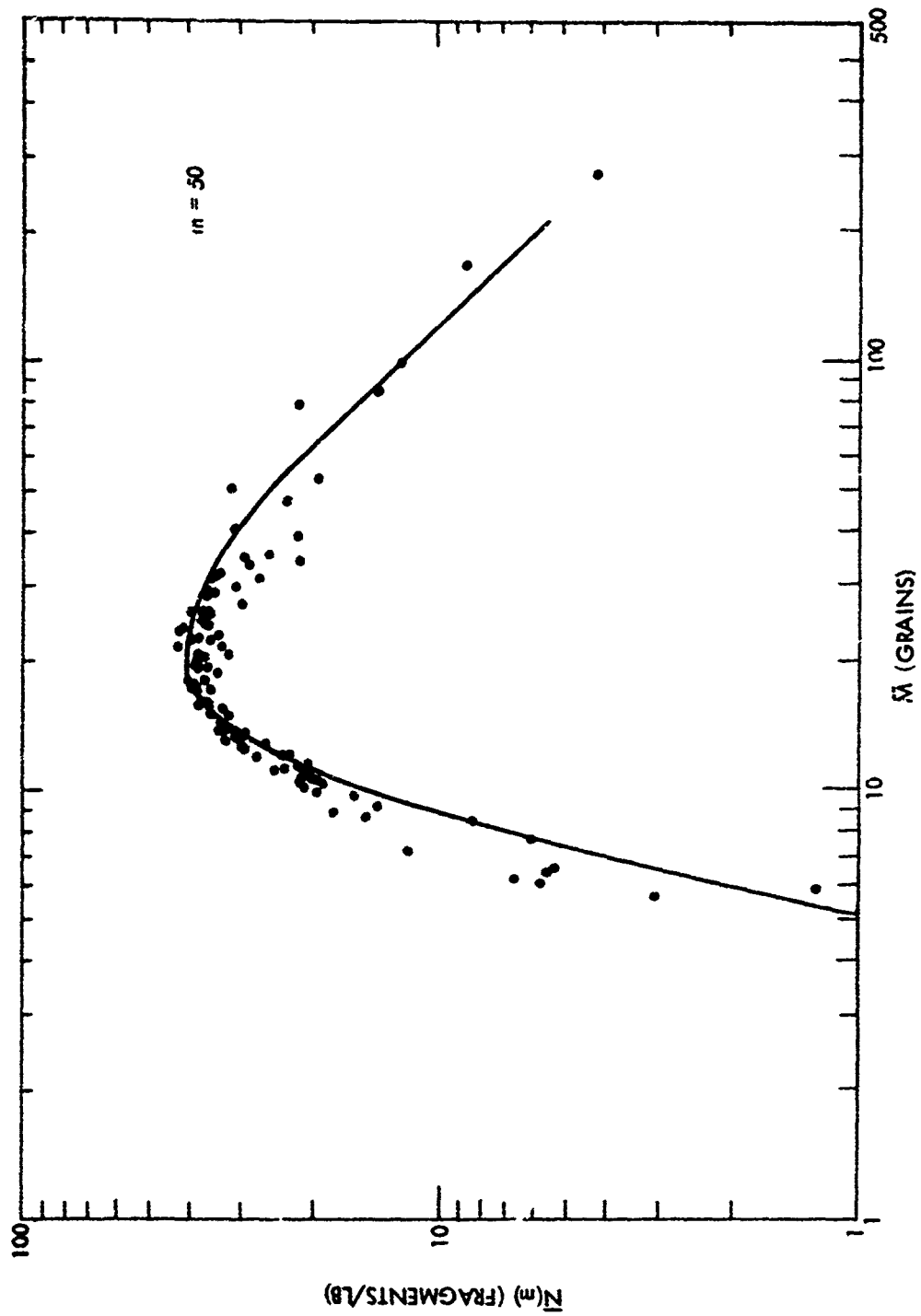


FIG. 6-5 THE NWL DATA (SUPPLEMENT TO REF 1), THE FITTED CURVES WERE CALCULATED WITH EQUATIONS (6) - (8). (CONT)

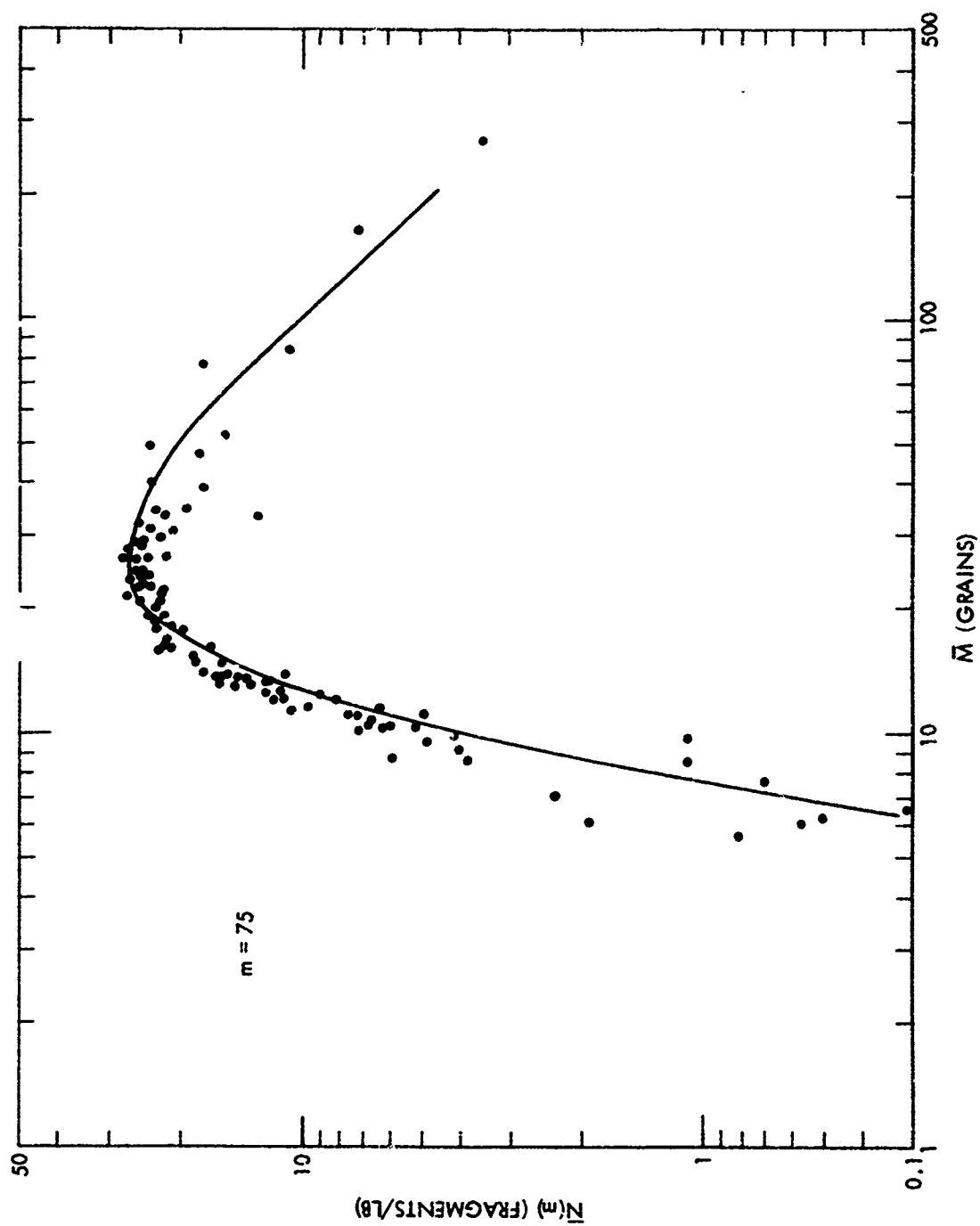


FIG. 6-6 THE NWL DATA (SUPPLEMENT TO REF 1). THE FITTED CURVES WERE CALCULATED WITH EQUATIONS (6) - (8). (CONT)

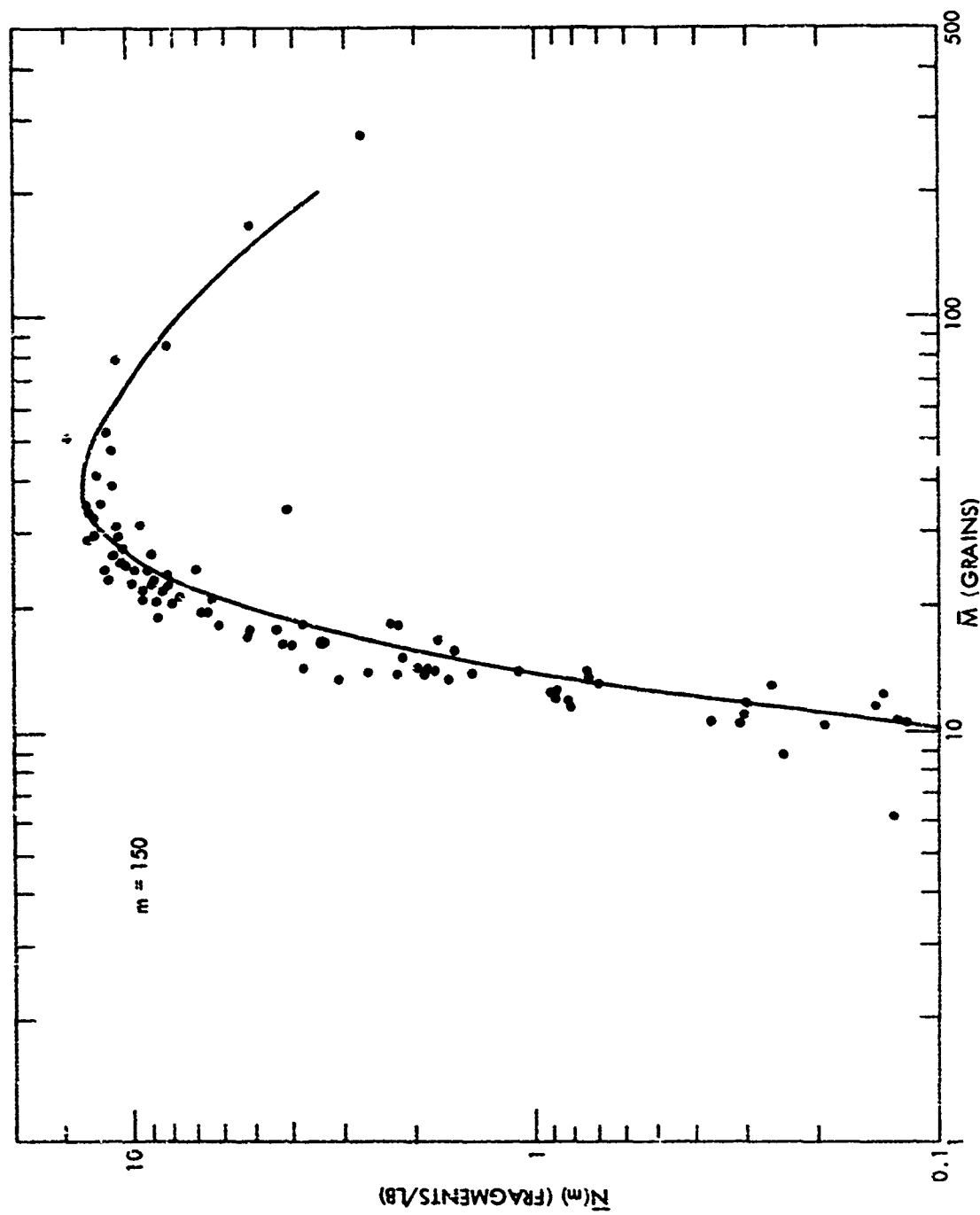


FIG. 6-7 THE NWL DATA (SUPPLEMENT TO REF 1). THE FITTED CURVES WERE CALCULATED WITH EQUATIONS (6) - (8). (CONT)

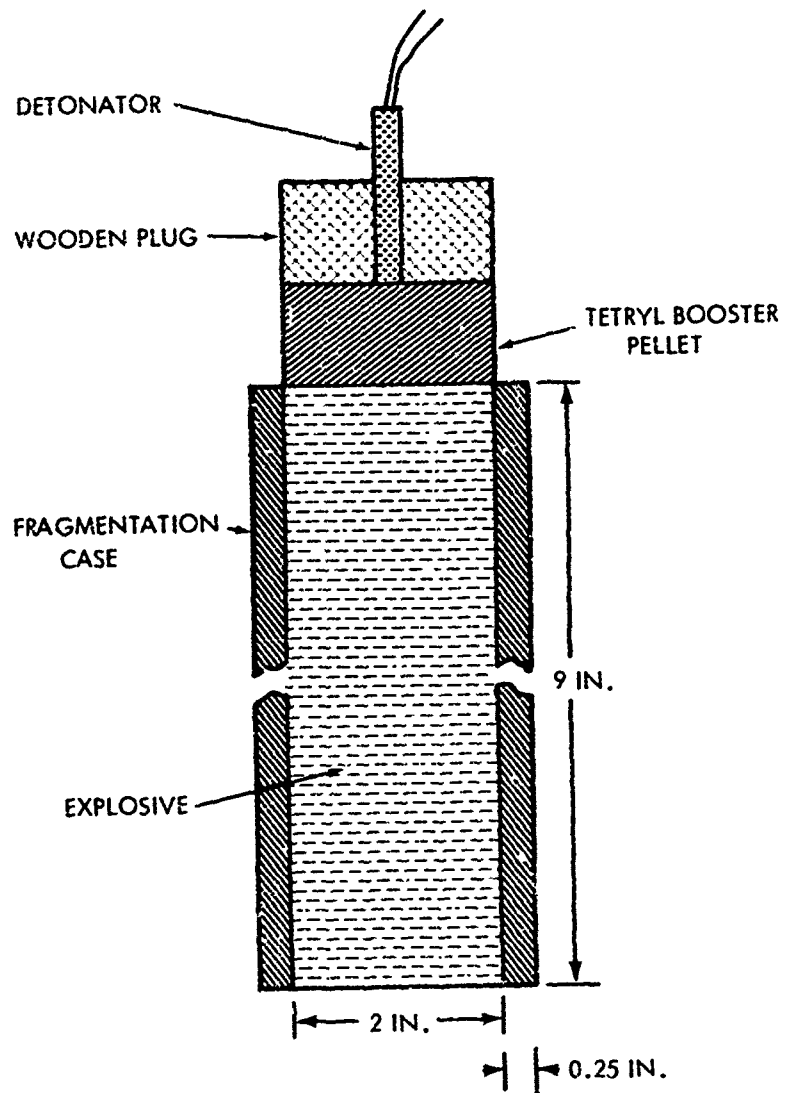


FIG. 7 ARRANGEMENT OF CHARGE AND INITIATOR FOR FIRING OF NOL CHARGES
(NAVORD REPORT 2933)

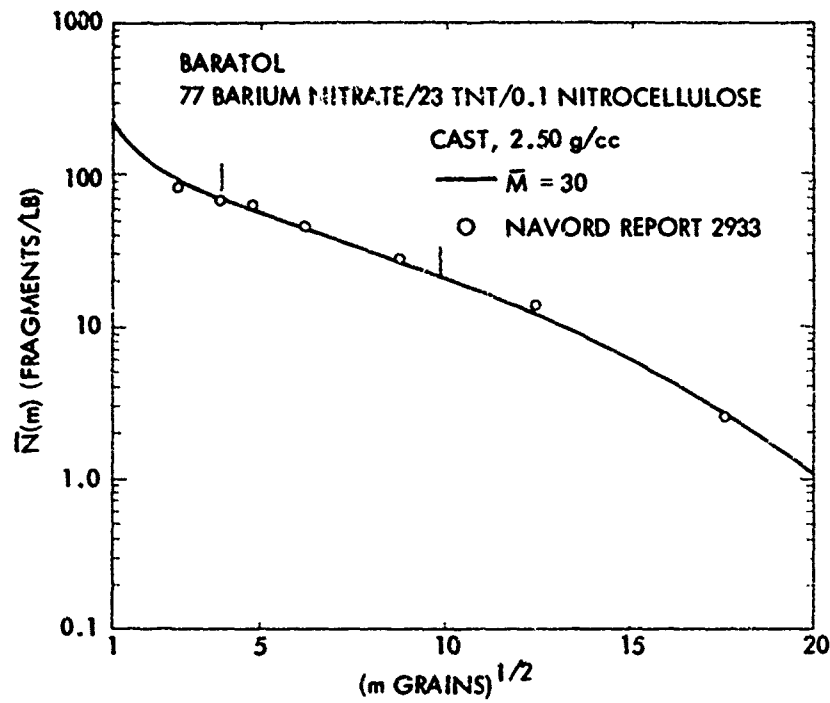


FIG. 8-1

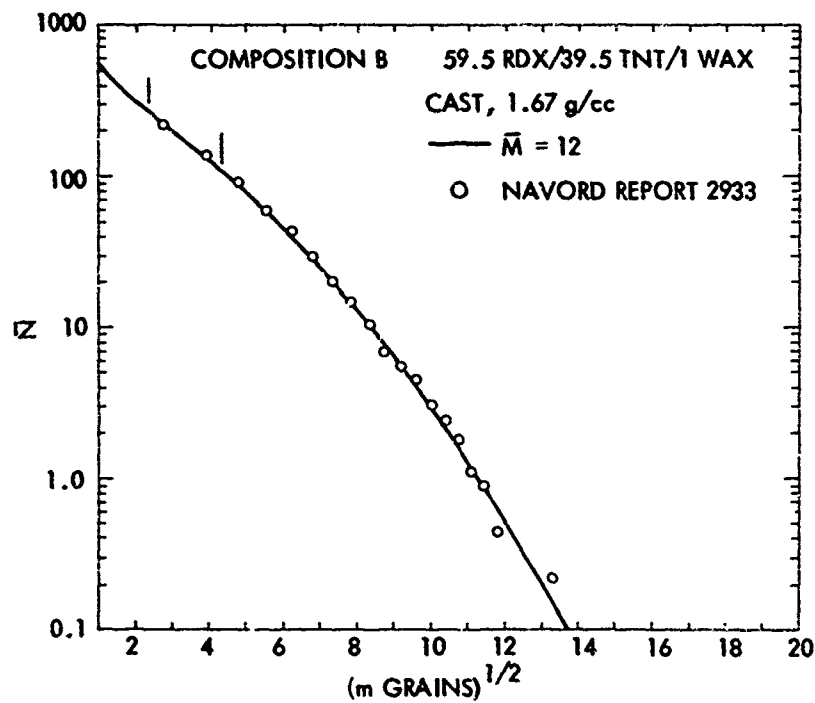


FIG. 8-2

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST)

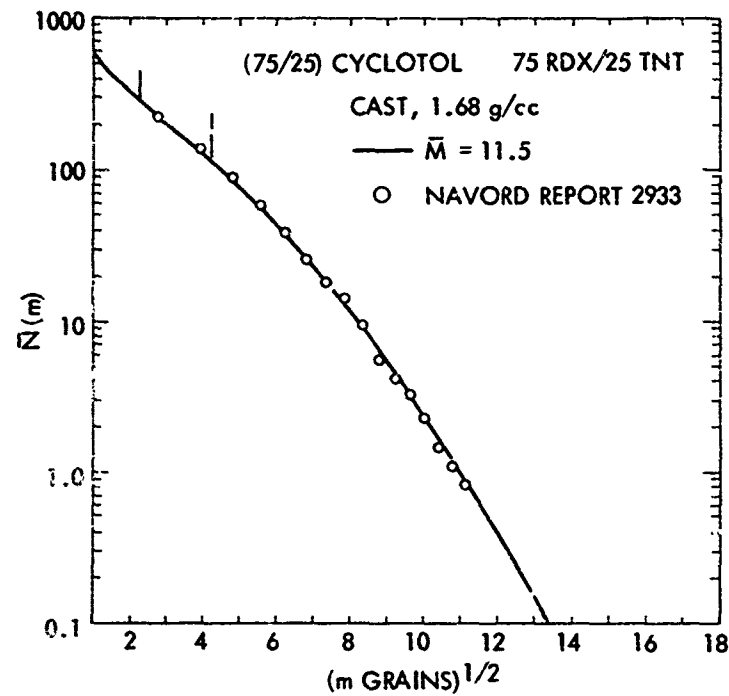


FIG. 8-3

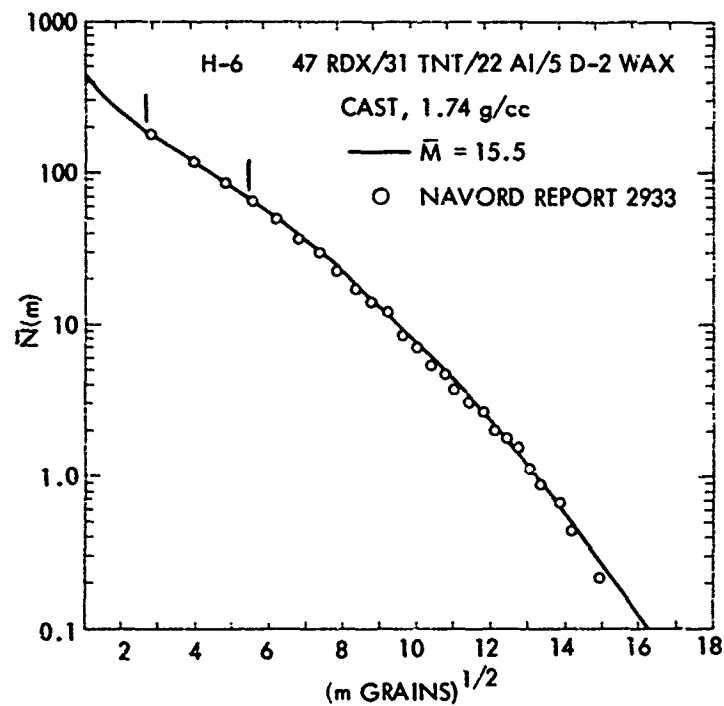


FIG. 8-4

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

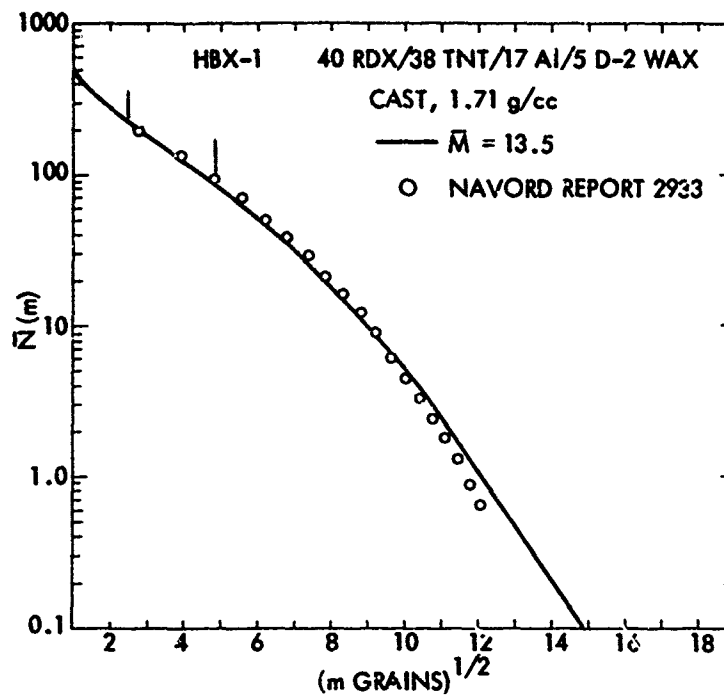


FIG. 8-5

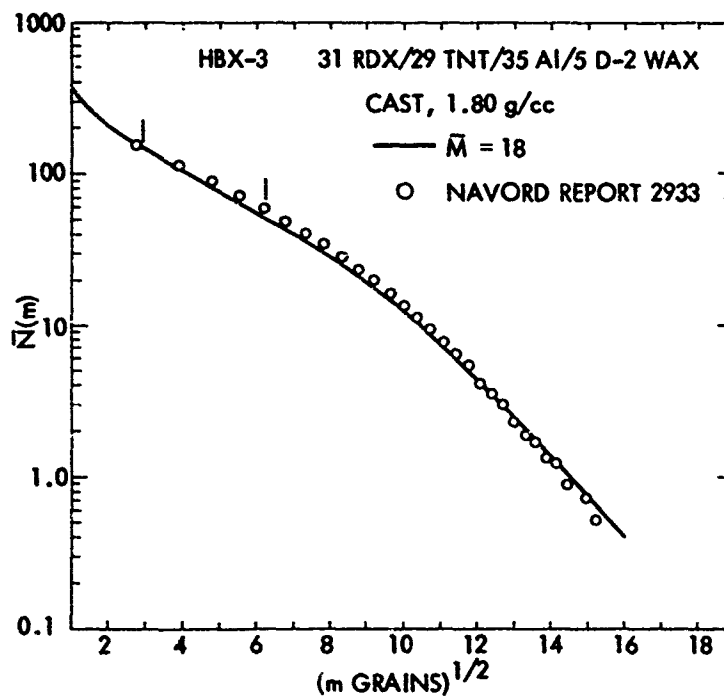


FIG. 8-6

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

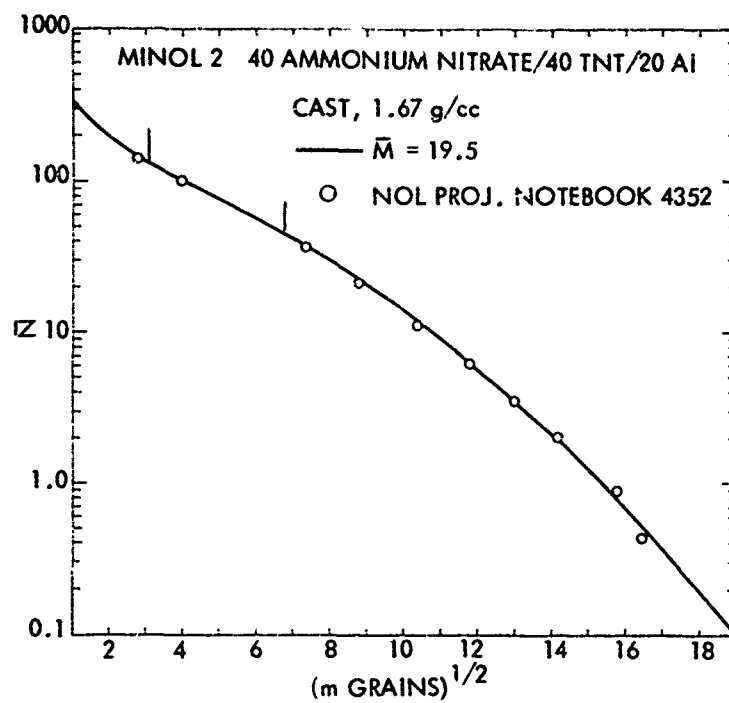


FIG. 8-7

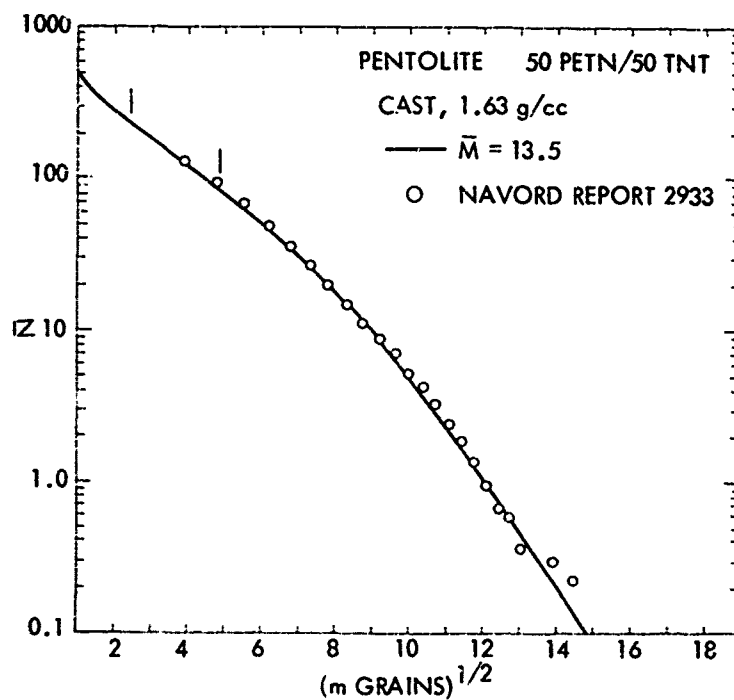


FIG. 8-8

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

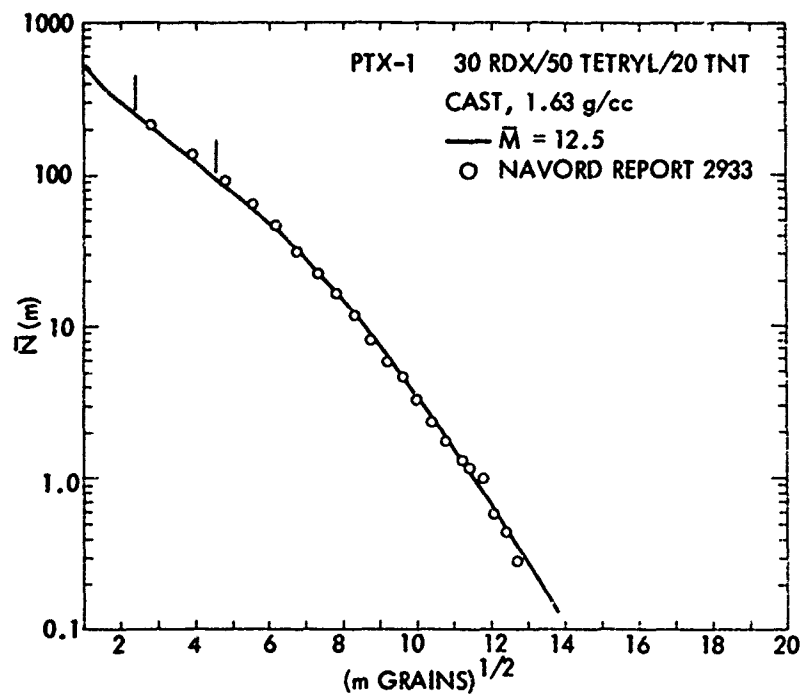


FIG. 8-9

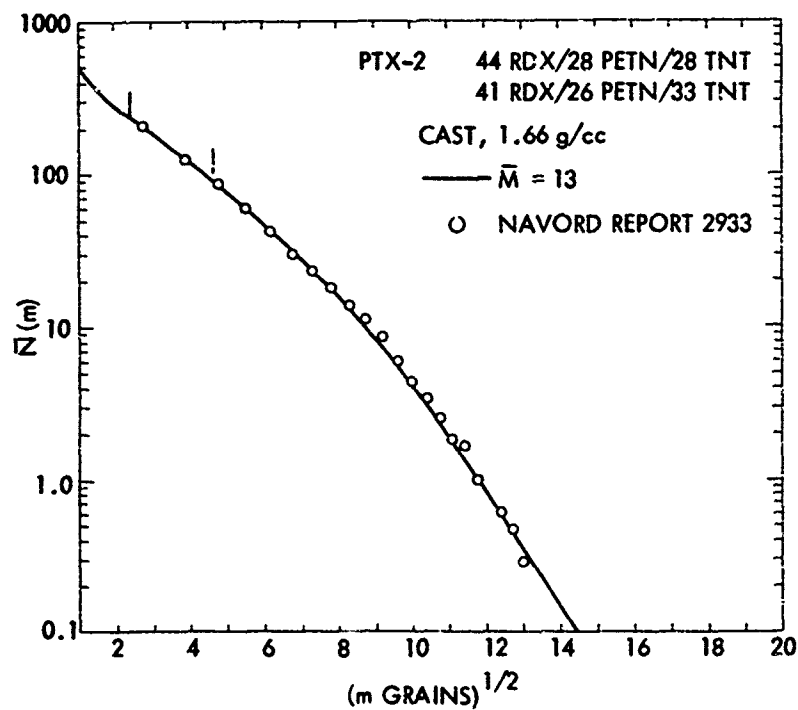


FIG. 8-10

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

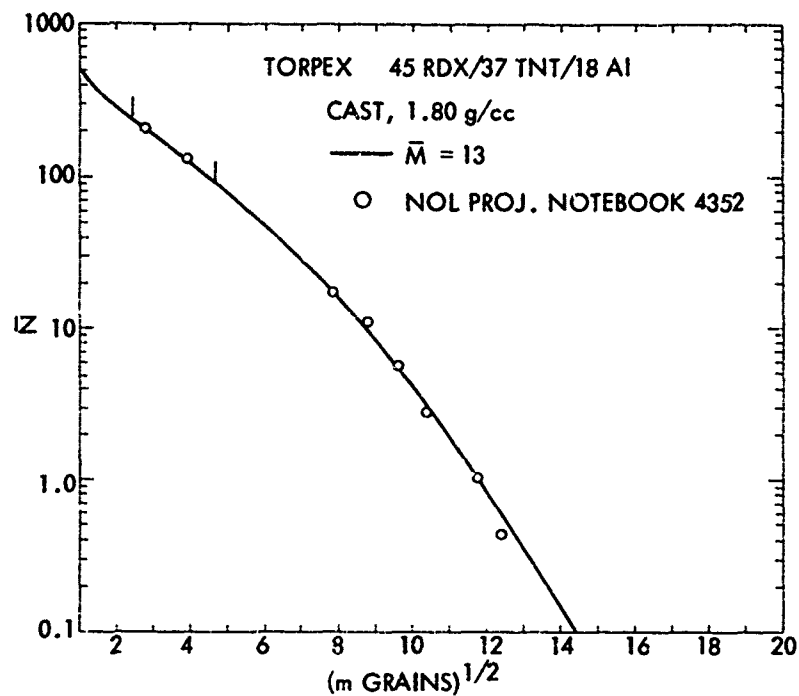


FIG. 8-11

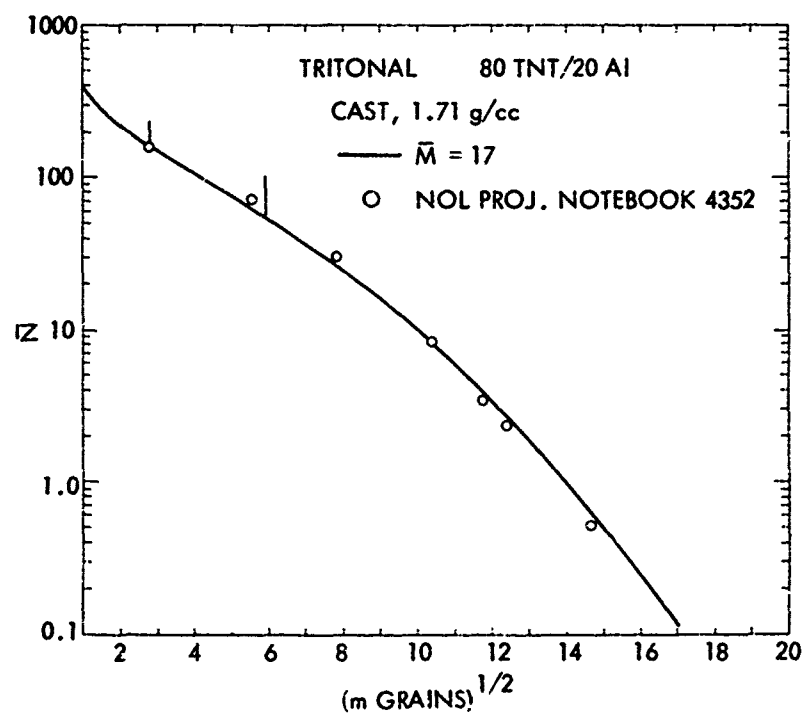


FIG. 8-12

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CCNT.)

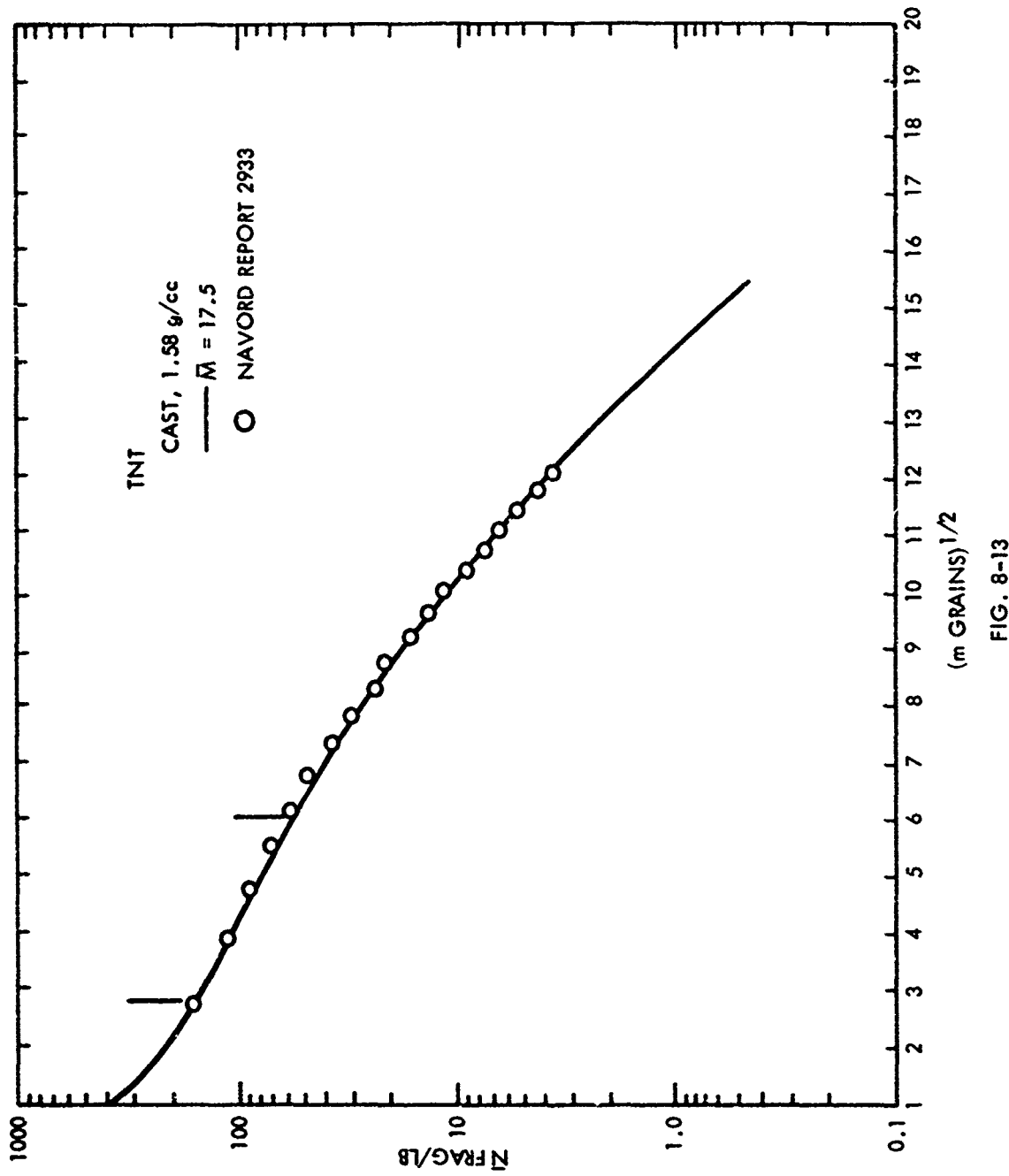


FIG. 8-13

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

NOLTR 73-83

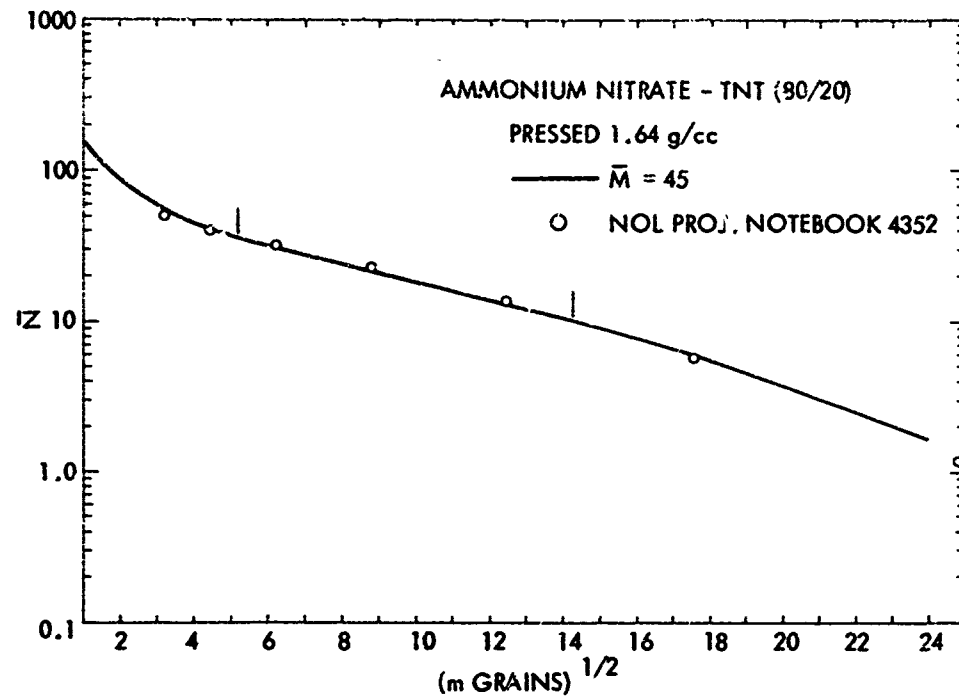


FIG. 8-14

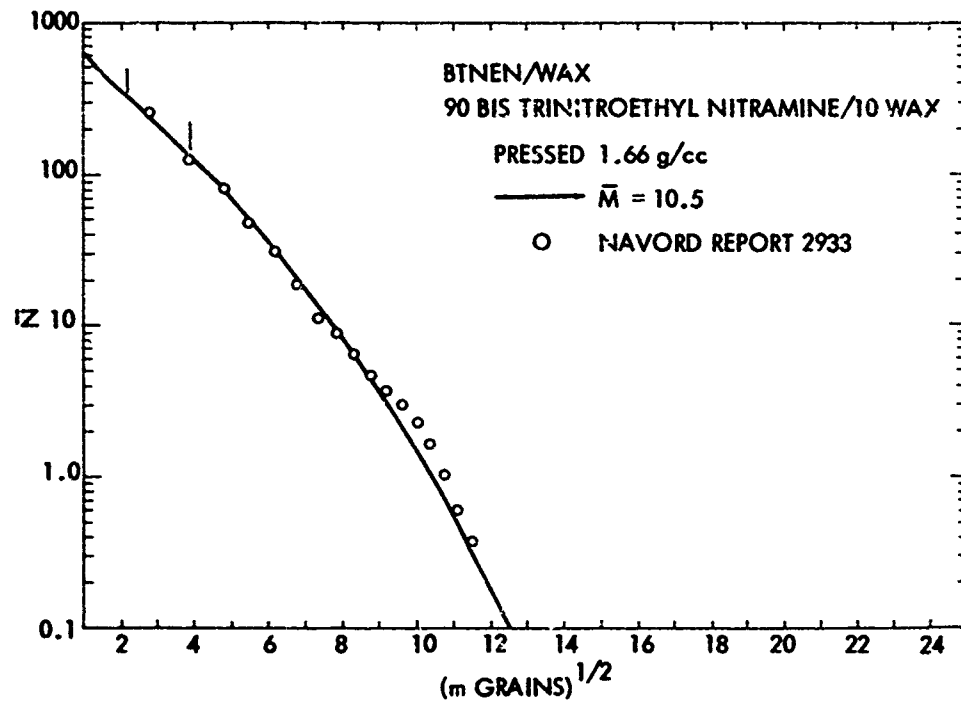


FIG. 8-15

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

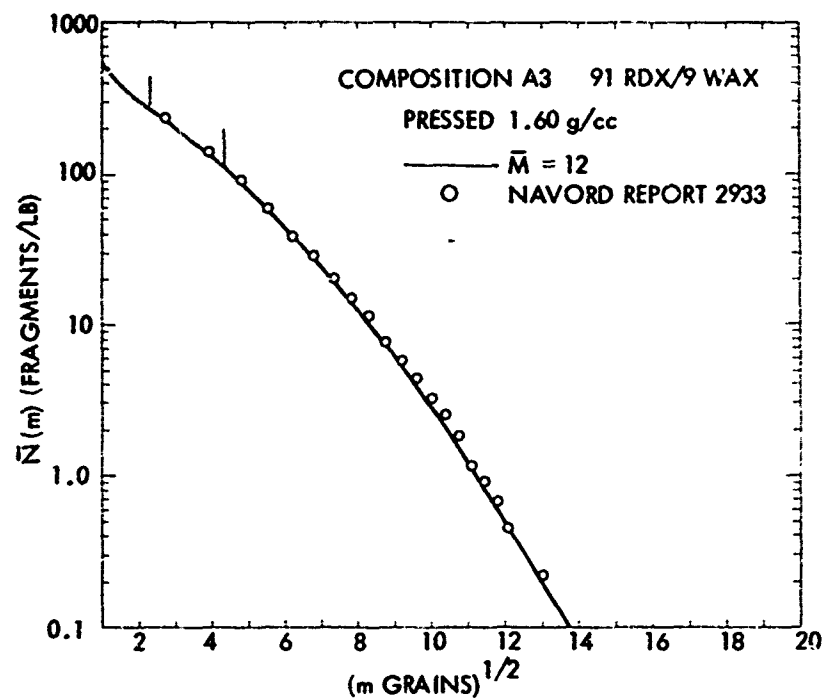


FIG. 8-16

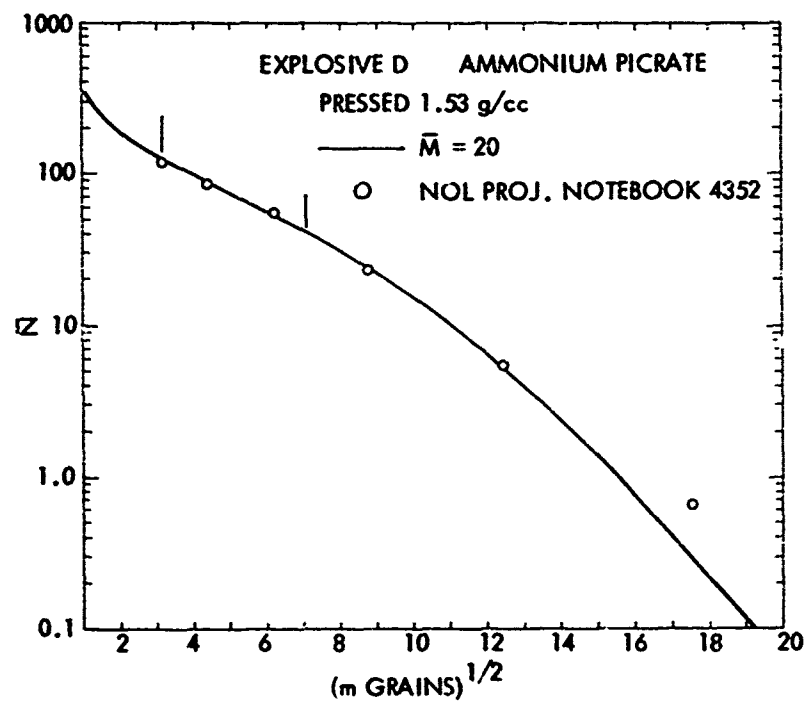


FIG. 8-17

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

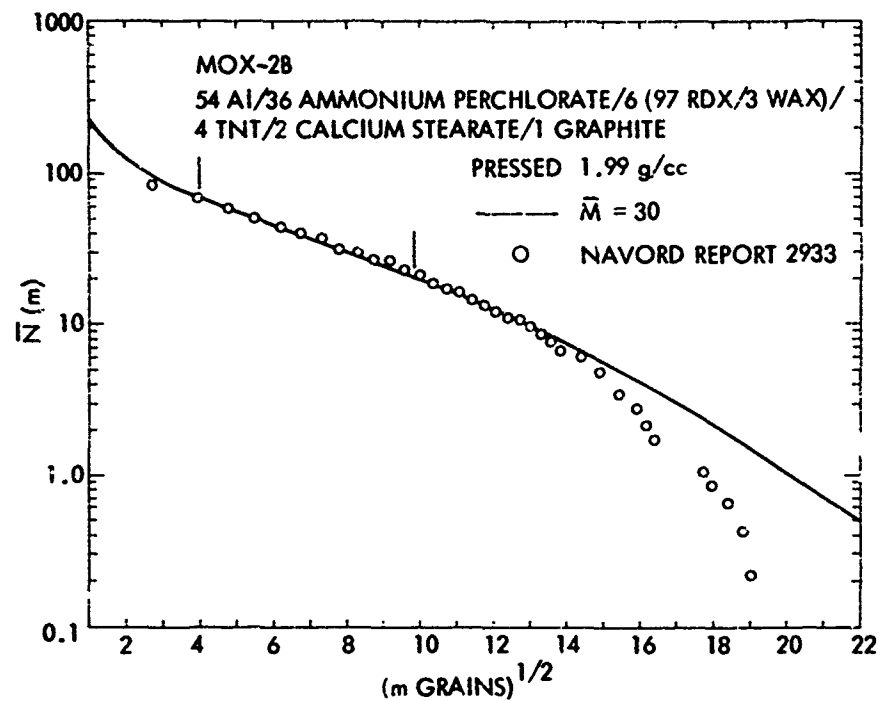


FIG. 8-18

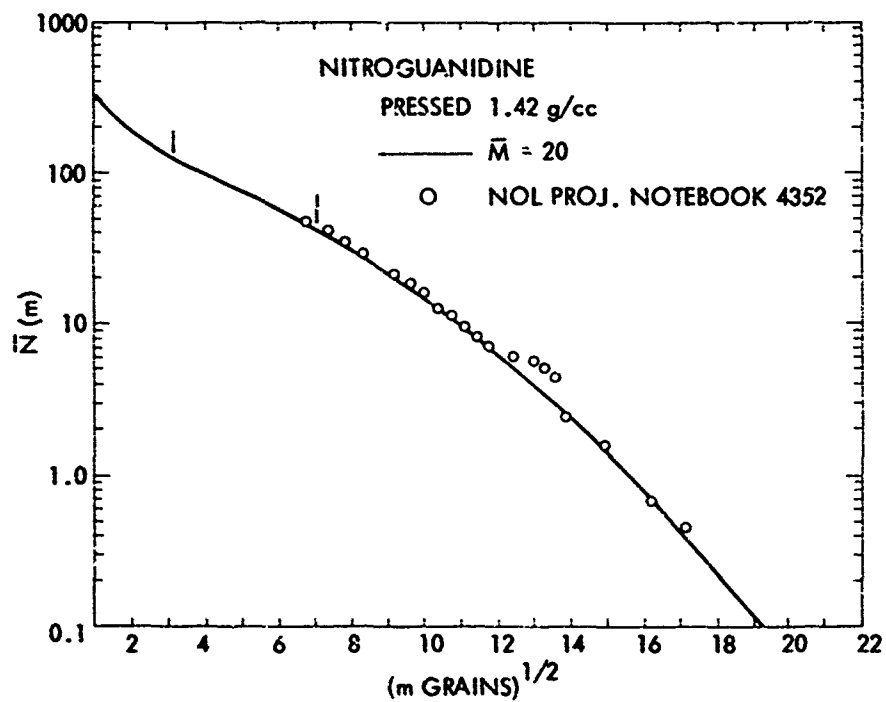


FIG. 8-19

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

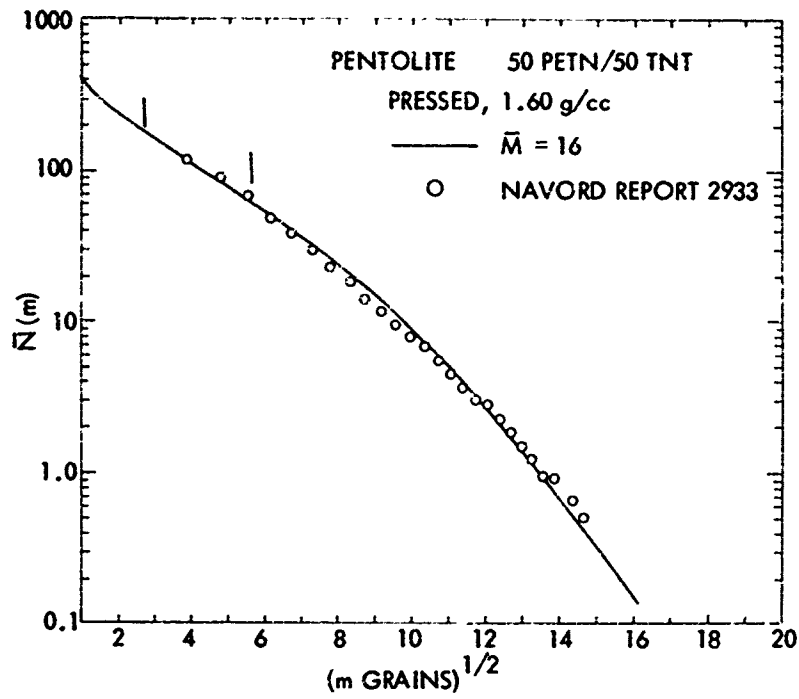


FIG. 8-20

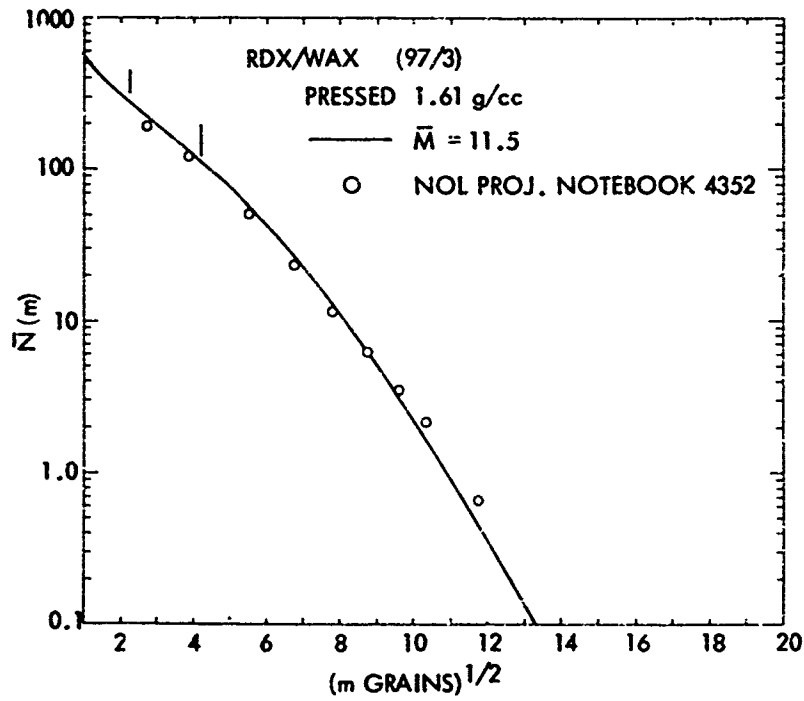


FIG. 8-21

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

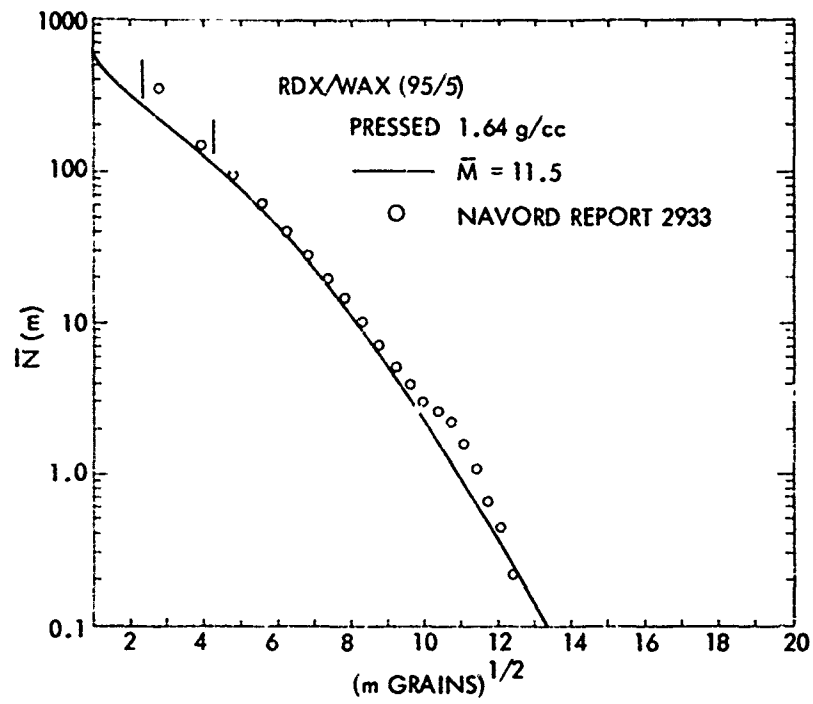


FIG. 8-22

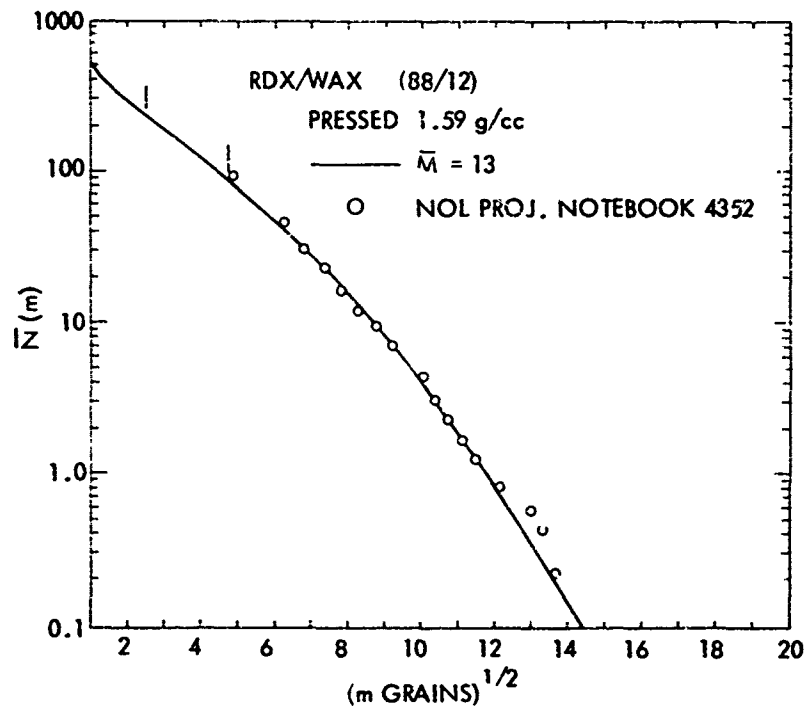


FIG. 8-23

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

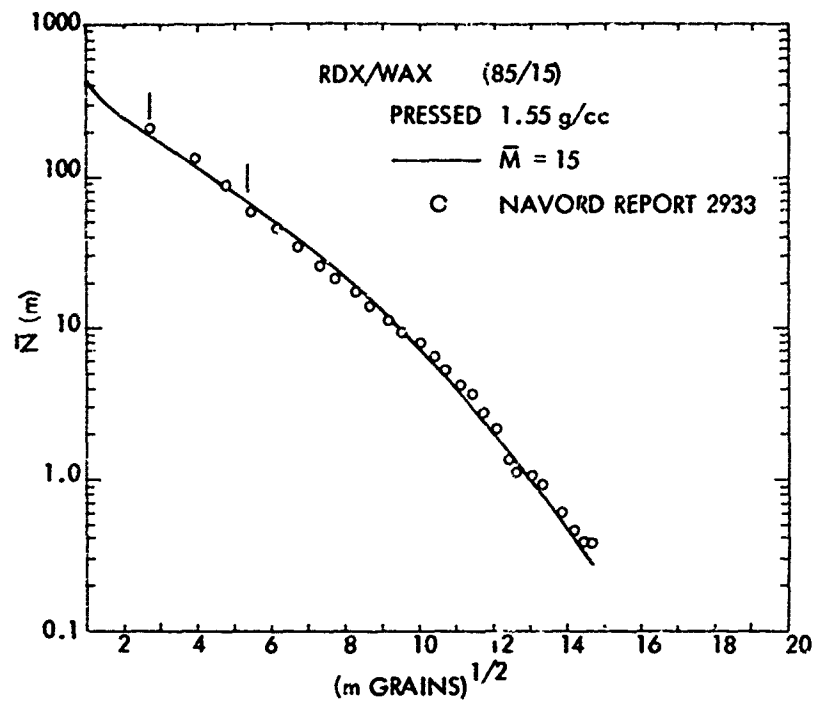


FIG. 8-24

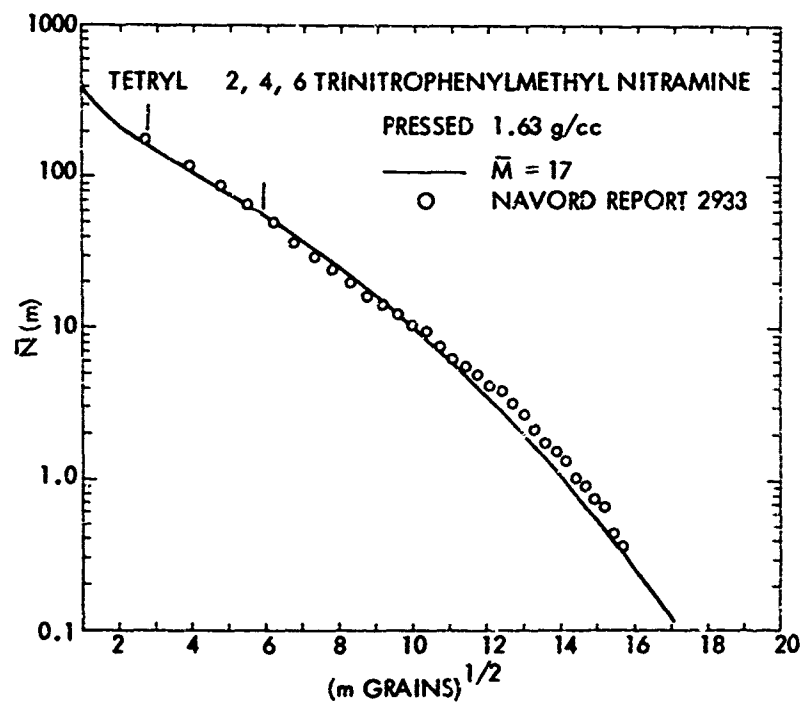


FIG. 8-25

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

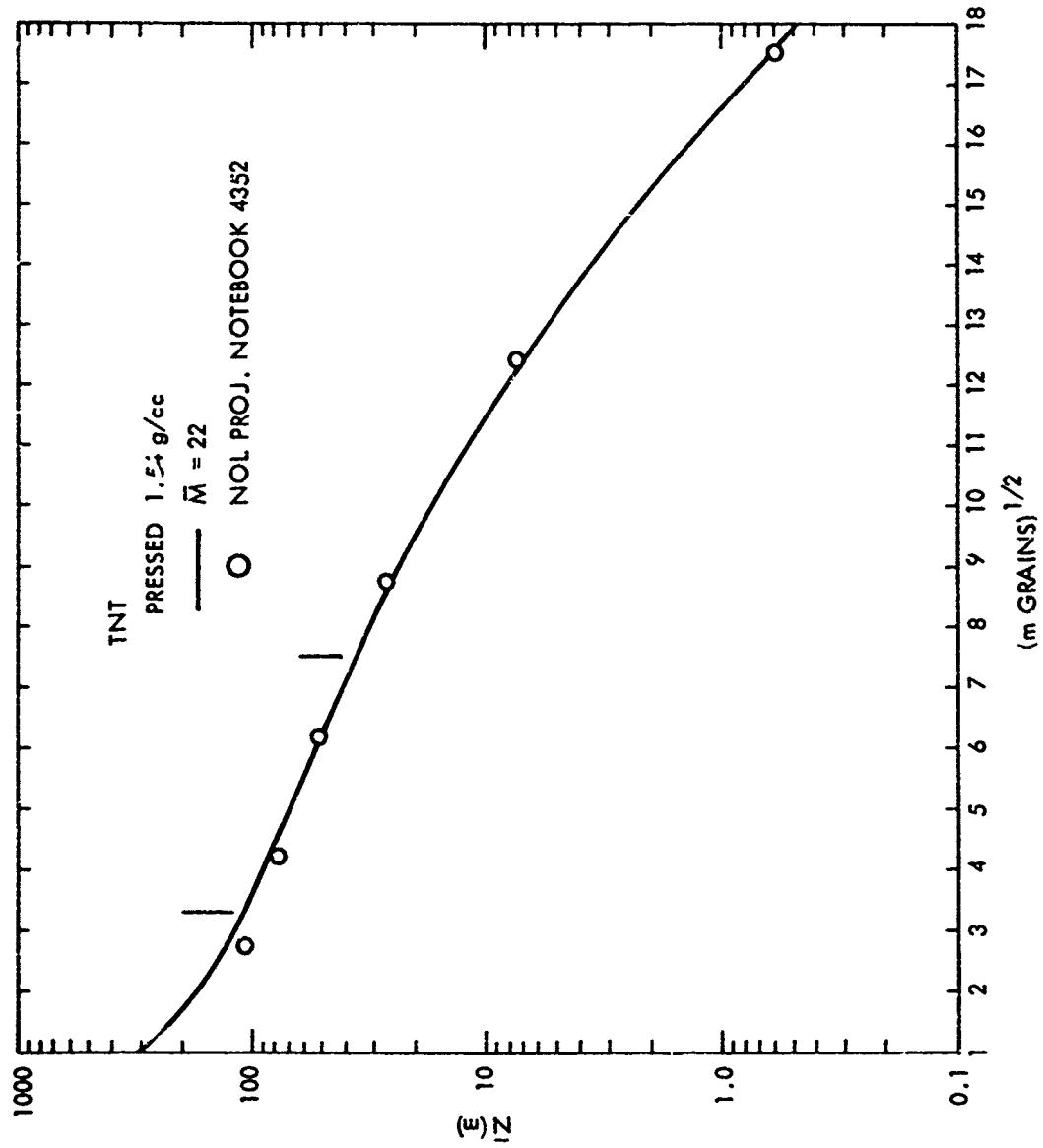


FIG. 8-26

FIG. 8 CUMULATIVE FRAGMENT WEIGHT DISTRIBUTION (2 IN. ID - 2.5 IN. OD CYLINDER FRAGMENTATION TEST) (CONT.)

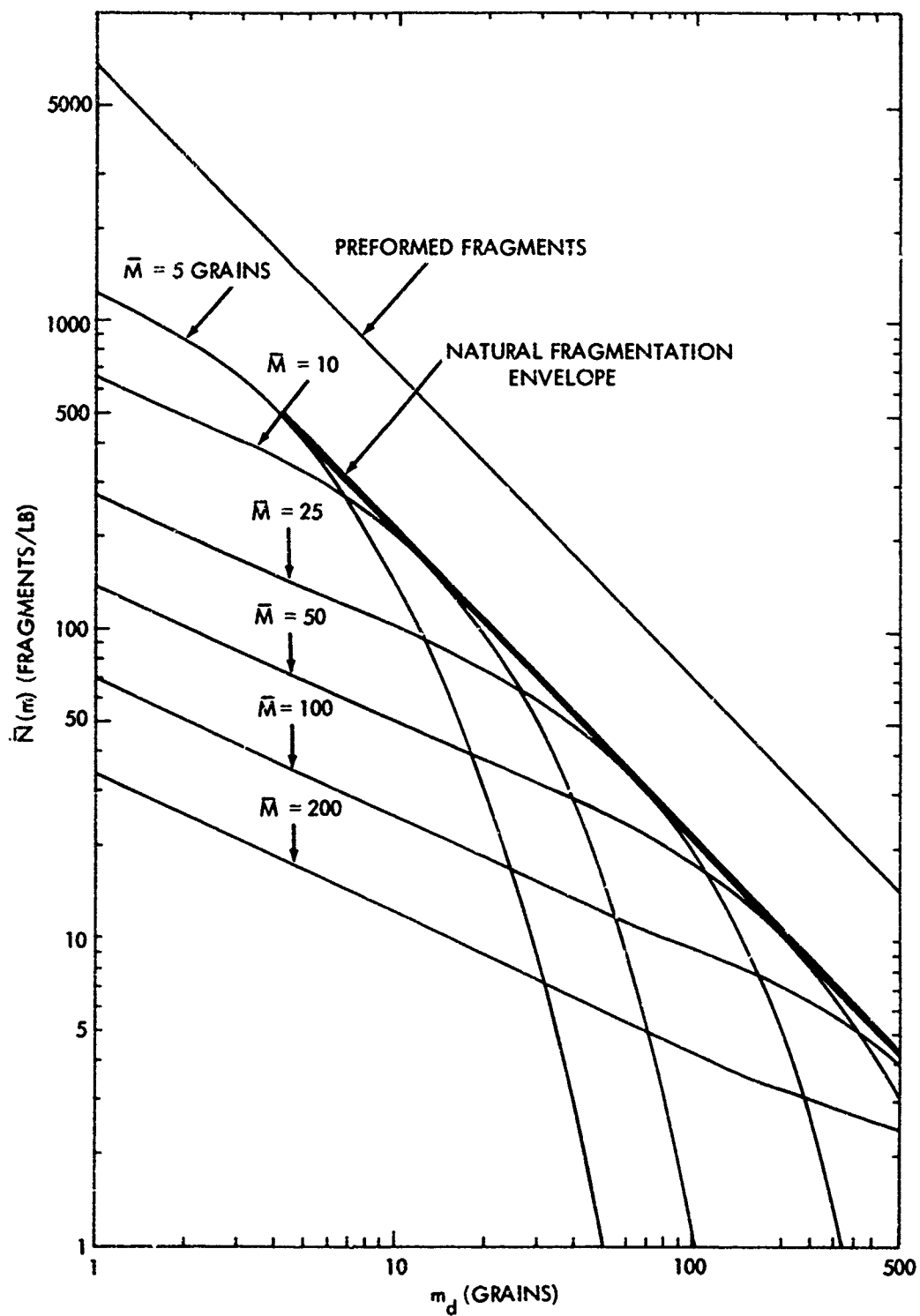


FIG. 9 CUMULATIVE FRAGMENTS PER LB. VS DESIGN WEIGHT

TABLE I

LIST-BASIC COMPUTER PROGRAM FOR FRAGMENT WEIGHT DISTRIBUTION CALCULATION

```

45   DIM M(30)
50   M(1)=1
52   M(2)=1.5
54   M(3)=2
56   M(4)=3
58   M(5)=4
60   M(6)=5
62   M(7)=7
64   M(8)=10
66   M(9)=20
68   M(10)=30
70   M(11)=50
72   M(12)=75
74   M(13)=100
76   M(14)=150
78   M(15)=200
80   M(16)=240
82   M(17)=480
95   REM FSQ034  CALCULATES NB VS MBAR
96   FOR K3=4 TO 50
97     C=0.2364
98     B2=1.77
99     L=2050/7000
100    M1=M3
110    B1=C/4
117    LET U2=B1*M1+B2
120    LET K=.6
140    LET M2=1+(2/9)*(M1-1)+(5/4)
153    LET M4=4*U2
155    M3=M4
160    N6=L/4/U2
162    N3=N6
164    N7=N3*EXP((M3/U2)**.5)
170    N2=N7*EXP(-(M2/U2)**.5)
190    LET N1=1/(M1+K)
200    LET P=LOG(N2/N1)/LOG(M2)
210    LET B=1+1/P
220    LET R1=N1*(1-(N2/N1)+B)/B
230    LET S1=N2*(M2+2*(M2*U2)+.5+2*U2)
240    LET S2=N3*(M3+2*(M3*U2)+.5+2*U2)
250    LET R2=S1-S2
260    LET R3=M1/(M1+K)-R1-R2
265    GOSUB 1000
267    U3=(2*Q*U2**.5*M3**(.Q-.5))**(1/Q)
269    N8=N3*EXP((M3/U3)**Q)
272    PRINT"MBAR="M1
275    PRINT"K="K,"N1B="L*7000
278    PRINT"ME="M4,"NEB="N6*7000
280    PRINT"M2="M2,"M3="M3
282    PRINT"SQR(M2)="SQR(M2),"SQR(M3)="SQR(M3)
285    PRINT"MU2="U2,"SQR(MU2)="U2+.5
287    PRINT"N2B="N2*7000,"N3B="N3*7000
288    PRINT "Q="Q
290    PRINT"MU3="U3
291    PRINT"ESN2="N7,"ESN3="N8
292    PRINT"W1/WT="R1,"W2/WT="R2,"W3/WT="R3

```

TABLE I (CONT.)

```

294 PRINT "(WT-W(1))/WT="K/(M1+K)
296 PRINT
298 PRINT"M", "SOR(M)", "NB"
300 FOR K2=1 TO 17
305 M=M(K2)
310 IF M<M2 THEN 500
320 IF M<M3 THEN 600
330 N=N8*EXP(-(M/U3)**0)
340 GO TO 700
500 LET N=N1*M*P
510 GO TO 700
600 N=N7*EXP(-(M/U2)**.5)
700 PRINT M, SOR(M), N*7000
710 NEXT K2
720 NEXT K3
725 GO TO 9999
1000 A=.2
1010 LET F4=0
1020 Q1=.5
1030 Q2=1.5
1040 LET R=Q1
1050 GOSUB 1500
1060 LET F1=F
1070 LET R=Q2
1080 GOSUB 1500
1090 LET F2=F
1100 LET Q3=Q1+(Q2-Q1)*ABS(F1)/(ABS(F1)+ABS(F2))
1110 GOSUB 1500
1120 LET F3=F
1130 IF ABS(F4-F3)<1E-7 THEN 1300
1134 IF ABS(F3)<1E-5 THEN 1300
1160 LET F4=F3
1170 PRINT
1180 LET Q5=Q3-A*(Q2-Q1)
1190 LET Q2=Q3+A*(Q2-Q1)
1200 LET Q1=Q5
1210 GO TO 1040
1300 LET Q=Q3
1305 PRINT
1310 RETURN
1500 LET U3=(2*R*U2+.5*M3*(R-.5))*(1/R)
1510 LET Y=EXP((M3/U3)*R)
1512 LET Y=Y*N3
1515 LET H=N3/Y/100
1520 S=0
1530 FOR J=1 TO 100 STEP 1
1540 F=(-LOG(J*H))*(1/R)
1550 IF J=100 THEN 1590
1560 IF ABS(INT(J/2)-J/2)<.1 THEN 1610
1570 LET S=S+4*F
1580 GO TO 1620
1590 LET S=S+F
1600 GO TO 1620
1610 LET S=S+2*F
1620 NEXT J
1630 LET F=H*S*Y*U3/3-R3
1640 RETURN
9999 END

```

TABLE II

Computer Program Outline

INPUT

Specify values of \bar{M} and m for which $\bar{N}(m)$ are to be calculated.
 Insert constants L , C , B_2 , and K .

CALCULATION PROCEDURE

→ For specified values of \bar{M}

Calculate listed parameters

| <u>Program Name</u> | <u>Text Name</u> | <u>Equation in Text</u> |
|---------------------|------------------------|-------------------------|
| U2 | μ_{II} | 19 |
| M2 | m_2 | 32 |
| M4 | $m_e (=m_3)$ | 18 |
| N6 | $N_e (=N_3)$ | 20 |
| N7 | n_{II} | 21 |
| N2 | N_2 | 12 |
| N1 | N_1 | 12 |
| P | p | 23 |
| R1, R2, R3 | W_I, W_{II}, W_{III} | 26-28 |
| Q | q | 24-29 |
| U3 | μ_{III} | 25 |
| N8 | n_{III} | 24 |

Print parameter values

→ For specified values of m

Calculate $\bar{N}(m)$

6, 7, or 8

Print $m, m^{1/2}, \bar{N}(m)$

→ Next m

→ Next \bar{M}

TABLE III
REGION BOUNDARY AND ENVELOPE POINTS FOR VARIOUS VALUES OF \bar{M}

| \bar{M} (GRAINS) | m_1 (GRAINS) | m_2 (GRAINS) | m_3 (GRAINS) | $N(m_3)$ (FRAGMENTS/GRAIN) |
|-----------------------|-------------------|-------------------|-------------------|-------------------------------|
| 4 | 1 | 1.87738 | 2.74975 | .106503 |
| 5 | 1 | 2.25708 | 4.08154 | 7.17516E-2 |
| 6 | 1 | 2.6615 | 5.63605 | 5.19614E-2 |
| 7 | 1 | 3.08678 | 7.40408 | 3.95535E-2 |
| 8 | 1 | 3.53023 | 9.37814 | 3.12276E-2 |
| 9 | 1 | 3.98985 | 11.552 | 2.53512E-2 |
| 10 | 1 | 4.4641 | 13.9203 | 2.10382E-2 |
| 11 | 1 | 4.95173 | 16.4783 | 1.77723E-2 |
| 12 | 1 | 5.45173 | 19.222 | 1.52355E-2 |
| 13 | 1 | 5.96323 | 22.1476 | .013223 |
| 14 | 1 | 6.48551 | 25.2519 | 1.15974E-2 |
| 15 | 1 | 7.01794 | 28.5318 | 1.02642E-2 |
| 16 | 1 | 7.55997 | 31.9845 | 9.15621E-3 |
| 17 | 1 | 8.11111 | 35.6075 | 8.22458E-3 |
| 18 | 1 | 8.67094 | 39.3985 | 7.43320E-3 |
| 19 | 1 | 9.23907 | 43.3552 | 6.75433E-3 |
| 20 | 1 | 9.81515 | 47.4756 | 6.16859E-3 |
| 22 | 1 | 10.9899 | 56.1999 | 5.21099E-3 |
| 24 | 1 | 12.193 | 65.5573 | 4.46719E-3 |
| 26 | 1 | 13.4226 | 75.5354 | 3.87709E-3 |
| 28 | 1 | 14.677 | 86.1226 | 3.40047E-3 |
| 30 | 1 | 15.955 | 97.3088 | 3.00957E-3 |
| 32 | 1 | 17.2551 | 109.084 | 2.68468E-3 |
| 34 | 1 | 18.5764 | 121.441 | 2.41152E-3 |
| 36 | 1 | 19.9179 | 134.37 | 2.17948E-3 |
| 38 | 1 | 21.2787 | 147.865 | 1.98058E-3 |
| 40 | 1 | 22.658 | 161.917 | 1.80862E-3 |
| 45 | 1 | 28.1828 | 199.45 | 1.46833E-3 |
| 50 | 1 | 33.1093 | 240.339 | 1.21852E-3 |
| 55 | 1 | 37.5297 | 284.505 | 1.02936E-3 |
| 60 | 1 | 41.3373 | 331.875 | 8.82431E-4 |
| 65 | 1 | 45.2265 | 382.388 | 7.65864E-4 |
| 70 | 1 | 49.1925 | 435.984 | 6.71715E-4 |
| 75 | 1 | 53.2311 | 492.613 | 5.94497E-4 |
| 80 | 1 | 57.3385 | 552.226 | 5.30321E-4 |
| 85 | 1 | 61.5115 | 614.779 | 4.76362E-4 |
| 90 | 1 | 65.747 | 680.231 | 4.30526E-4 |
| 95 | 1 | 69.0425 | 748.546 | 3.91235E-4 |
| 100 | 1 | 72.3955 | 819.686 | 3.57280E-4 |
| 110 | 1 | 79.2655 | 970.315 | 3.01817E-4 |
| 120 | 1 | 88.3416 | 1131.87 | 2.58736E-4 |
| 130 | 1 | 97.6106 | 1304.15 | 2.24558E-4 |
| 140 | 1 | 107.061 | 1486.94 | 1.96953E-4 |
| 150 | 1 | 116.683 | 1680.08 | 1.74312E-4 |
| 160 | 1 | 126.468 | 1883.39 | 1.55495E-4 |
| 170 | 1 | 136.408 | 2096.73 | 1.39673E-4 |
| 180 | 1 | 146.497 | 2319.96 | 1.26234E-4 |
| 190 | 1 | 156.727 | 2552.94 | 1.14713E-4 |
| 200 | 1 | 167.094 | 2795.57 | 1.04758E-4 |

NOLTR 73-83

TABLE IV
CALCULATED VALUES OF PARAMETERS p , n_{II} , μ_{II} , AND N_2

| \bar{M} (GRAINS) | p | n_{II} (GRAINS) | μ_{II} (GRAINS) | N_2 (FRAGMENTS/GRAIN) |
|-----------------------|----------|----------------------|------------------------|----------------------------|
| 4 | -.531214 | .786957 | .687438 | .150747 |
| 5 | -.490195 | .530176 | 1.02039 | .119813 |
| 6 | -.454148 | .383946 | 1.40901 | 9.71372E-2 |
| 7 | -.437672 | .292263 | 1.85102 | 8.03421E-2 |
| 8 | -.429512 | .230743 | 2.34454 | 6.76416E-2 |
| 9 | -.425328 | .187322 | 2.888 | 5.78262E-2 |
| 10 | -.423212 | .155452 | 3.43007 | 5.00863E-2 |
| 11 | -.422237 | .13132 | 4.11958 | 4.38723E-2 |
| 12 | -.421919 | .112576 | 4.8055 | 3.88036E-2 |
| 13 | -.421993 | 9.77052E-2 | 5.53691 | .034611 |
| 14 | -.422302 | 8.56941E-2 | 6.31298 | 3.11001E-2 |
| 15 | -.422754 | .075843 | 7.13295 | 2.81279E-2 |
| 16 | -.423292 | 6.76558E-2 | 7.99613 | 2.55871E-2 |
| 17 | -.42388 | 6.07719E-2 | 8.90189 | 2.33963E-2 |
| 18 | -.424494 | 5.49244E-2 | 9.84963 | 2.14923E-2 |
| 19 | -.425119 | 4.99118E-2 | 10.8388 | 1.98261E-2 |
| 20 | -.425745 | .04558 | 11.8689 | 1.83585E-2 |
| 22 | -.426978 | 3.85043E-2 | 14.05 | 1.59005E-2 |
| 24 | -.428164 | 3.30083E-2 | 16.3893 | 1.39326E-2 |
| 26 | -.429293 | .028648 | 18.8938 | 1.23295E-2 |
| 28 | -.430362 | 2.51262E-2 | 21.5307 | 1.10042E-2 |
| 30 | -.431373 | 2.22378E-2 | 24.3272 | 9.89422E-3 |
| 32 | -.432329 | 1.98373E-2 | 27.2711 | 8.95420E-3 |
| 34 | -.433234 | 1.78188E-2 | 30.3602 | 8.15016E-3 |
| 36 | -.434092 | 1.61043E-2 | 33.5925 | 7.45638E-3 |
| 38 | -.434906 | 1.46346E-2 | 36.9662 | 6.85301E-3 |
| 40 | -.435681 | 1.33645E-2 | 40.4794 | 6.32455E-3 |
| 45 | -.437464 | 1.08495E-2 | 49.8625 | 5.25660E-3 |
| 50 | -.439059 | 9.00369E-3 | 60.0848 | 4.45165E-3 |
| 55 | -.440501 | 7.60598E-3 | 71.1262 | .003828 |
| 60 | -.441315 | 6.52033E-3 | 82.9689 | 3.33373E-3 |
| 65 | -.44302 | 5.65901E-3 | 95.597 | 2.93454E-3 |
| 70 | -.444133 | 4.96334E-3 | 108.996 | 2.60691E-3 |
| 75 | -.445166 | 4.39277E-3 | 123.153 | 2.33427E-3 |
| 80 | -.446129 | 3.91857E-3 | 138.056 | 2.10466E-3 |
| 85 | -.447031 | 3.51986E-3 | 153.695 | 1.90925E-3 |
| 90 | -.447978 | 3.18118E-3 | 170.058 | 1.74138E-3 |
| 95 | -.448678 | 2.89036E-3 | 187.136 | 1.59598E-3 |
| 100 | -.449434 | 2.63996E-3 | 204.921 | 1.46911E-3 |
| 110 | -.450832 | 2.23014E-3 | 242.579 | 1.25915E-3 |
| 120 | -.452102 | 1.91182E-3 | 282.969 | 1.09342E-3 |
| 130 | -.453264 | 1.65927E-3 | 326.037 | 9.60039E-4 |
| 140 | -.454335 | 1.45529E-3 | 371.736 | 8.50908E-4 |
| 150 | -.455326 | .001288 | 420.019 | 7.60346E-4 |
| 160 | -.45625 | 1.14896E-3 | 470.847 | 6.84267E-4 |
| 170 | -.457113 | 1.03205E-3 | 524.182 | 6.19665E-4 |
| 180 | -.457924 | 9.32749E-4 | 579.989 | 5.64284E-4 |
| 190 | -.458687 | 8.47624E-4 | 638.236 | 5.16407E-4 |
| 200 | -.459409 | 7.74059E-4 | 698.893 | 4.74702E-4 |

TABLE V
CALCULATED VALUES OF PARAMETERS n_{III} , μ_{III} , AND q

| \bar{M} (GRAINS) | n_{III} (GRAINS) | μ_{III} (GRAINS) | q |
|-----------------------|-----------------------|-------------------------|---------|
| 4 | .588015 | 1.10108 | .585282 |
| 5 | .338579 | 2.06463 | .644517 |
| 6 | .219969 | 3.32018 | .693007 |
| 7 | .154941 | 4.8394 | .73239 |
| 8 | .115506 | 6.60063 | .764517 |
| 9 | 8.97608E-2 | 8.58769 | .790938 |
| 10 | 7.19925E-2 | 10.7882 | .812861 |
| 11 | 5.91875E-2 | 13.1922 | .831207 |
| 12 | 4.96358E-2 | 15.7917 | .846681 |
| 13 | 4.23081E-2 | 18.58 | .859827 |
| 14 | 3.65545E-2 | 21.5511 | .871064 |
| 15 | 3.19476E-2 | 24.7 | .880722 |
| 16 | 2.81969E-2 | 28.0221 | .889065 |
| 17 | 2.50989E-2 | 31.5133 | .8963 |
| 18 | .022508 | 35.1699 | .902598 |
| 19 | 2.03171E-2 | 38.9885 | .908097 |
| 20 | 1.84464E-2 | 42.9659 | .91291 |
| | | | |
| 24 | 1.54365E-2 | 51.3856 | .920841 |
| 26 | 1.31384E-2 | 60.408 | .926974 |
| 28 | 1.13403E-2 | 70.0146 | .931727 |
| 30 | 9.90431E-3 | 80.1887 | .935403 |
| 32 | 8.73758E-3 | 90.9155 | .938229 |
| 34 | 7.77541E-3 | 102.181 | .940377 |
| 36 | 6.97167E-3 | 113.974 | .941976 |
| 38 | 6.29266E-3 | 126.282 | .943131 |
| 40 | 5.71330E-3 | 139.095 | .943922 |
| | | | |
| 45 | 5.21457E-3 | 152.403 | .944413 |
| | | | |
| 45 | 4.23220E-3 | 187.781 | .944644 |
| 50 | 3.51519E-3 | 226.073 | .943877 |
| 55 | 2.97422E-3 | 267.168 | .942466 |
| 60 | 2.55494E-3 | 310.972 | .94064 |
| 65 | 2.22271E-3 | 357.402 | .938548 |
| 70 | 1.95448E-3 | 406.383 | .936289 |
| 75 | 1.73447E-3 | 457.847 | .933931 |
| 80 | 1.55153E-3 | 511.735 | .931521 |
| 85 | 1.39758E-3 | 567.99 | .929093 |
| 90 | 1.26666E-3 | 626.562 | .92667 |
| 95 | 1.15430E-3 | 687.405 | .924266 |
| 100 | 1.05706E-3 | 750.475 | .921894 |
| | | | |
| 110 | 8.97855E-4 | 883.136 | .917273 |
| 120 | 7.73783E-4 | 1024.26 | .91284 |
| 130 | 6.75003E-4 | 1173.59 | .908609 |
| 140 | 5.94933E-4 | 1330.9 | .90458 |
| 150 | 5.29026E-4 | 1495.99 | .900746 |
| 160 | 4.74052E-4 | 1668.67 | .897099 |
| 170 | 4.27666E-4 | 1848.78 | .893627 |
| 180 | 3.88125E-4 | 2036.16 | .890321 |
| 190 | 3.54114E-4 | 2230.67 | .887169 |
| 200 | 3.24623E-4 | 2432.18 | .884161 |

TABLE VI
CALCULATED WEIGHT FRACTIONS IN VARIOUS REGIONS

| \bar{M} (GRAINS) | 1-W(I) | W_I | W_{II} | W_{III} |
|-----------------------|------------|------------|----------|-----------|
| 4 | .130435 | 9.10696E-2 | .100636 | .67786 |
| 5 | .107143 | 8.83233E-2 | .146453 | .658081 |
| 6 | 9.09091E-2 | 8.90368E-2 | .176338 | .643716 |
| 7 | 7.89474E-2 | 9.06119E-2 | .197374 | .633067 |
| 8 | 6.97674E-2 | 9.22368E-2 | .213024 | .624972 |
| 9 | .0625 | 9.36638E-2 | .225162 | .618674 |
| 10 | 5.66038E-2 | 9.48362E-2 | .234885 | .613675 |
| 11 | 5.17241E-2 | 9.57633E-2 | .242872 | .609641 |
| 12 | .047619 | 9.64741E-2 | .24957 | .606337 |
| 13 | 4.41176E-2 | 9.70013E-2 | .255283 | .603599 |
| 14 | 4.10959E-2 | 9.73752E-2 | .260223 | .601306 |
| 15 | 3.84615E-2 | .097622 | .264547 | .59937 |
| 16 | 3.61446E-2 | 9.77638E-2 | .26837 | .597722 |
| 17 | 3.40909E-2 | 9.78189E-2 | .271779 | .596311 |
| 18 | 3.22581E-2 | 9.78025E-2 | .274843 | .595097 |
| 19 | 3.06122E-2 | 9.77267E-2 | .277615 | .594046 |
| 20 | 2.91262E-2 | .097602 | .280137 | .593135 |
| 22 | 2.65437E-2 | 9.72377E-2 | .284566 | .591748 |
| 24 | 2.43902E-2 | 9.67611E-2 | .288338 | .590511 |
| 26 | 2.25564E-2 | 9.62081E-2 | .291598 | .589637 |
| 28 | .020979 | 9.56038E-2 | .294451 | .588966 |
| 30 | 1.96078E-2 | .094966 | .296973 | .588453 |
| 32 | 1.84049E-2 | 9.43076E-2 | .299223 | .588064 |
| 34 | .017341 | 9.36379E-2 | .301246 | .587776 |
| 36 | 1.63934E-2 | 9.29636E-2 | .303075 | .587568 |
| 38 | .015544 | 9.22897E-2 | .304741 | .587425 |
| 40 | 1.47783E-2 | 9.16197E-2 | .306265 | .587337 |
| 45 | 1.31579E-2 | 8.99775E-2 | .309569 | .587296 |
| 50 | 1.18577E-2 | 8.83987E-2 | .31231 | .587434 |
| 55 | 1.07914E-2 | 8.68926E-2 | .314629 | .587687 |
| 60 | 9.90099E-3 | 8.54613E-2 | .316621 | .588017 |
| 65 | 9.14624E-3 | 8.41029E-2 | .318356 | .588395 |
| 70 | 8.49858E-3 | 8.28142E-2 | .319882 | .588805 |
| 75 | 7.93651E-3 | 8.15909E-2 | .321238 | .589234 |
| 80 | 7.44417E-3 | 8.04286E-2 | .322452 | .589675 |
| 85 | 7.00935E-3 | 7.93231E-2 | .323547 | .590121 |
| 90 | 6.52252E-3 | 7.82702E-2 | .32454 | .590567 |
| 95 | 6.27615E-3 | 7.72661E-2 | .325446 | .591012 |
| 100 | 5.96421E-3 | 7.63073E-2 | .326277 | .591452 |
| 110 | 5.42495E-3 | 7.45127E-2 | .327748 | .592315 |
| 120 | 4.97512E-3 | 7.28636E-2 | .329013 | .593148 |
| 130 | 4.59418E-3 | 7.13411E-2 | .330116 | .593949 |
| 140 | 4.26743E-3 | 6.99295E-2 | .331086 | .594717 |
| 150 | 3.98406E-3 | 6.86154E-2 | .331948 | .595453 |
| 160 | 3.73599E-3 | 6.73877E-2 | .33272 | .596157 |
| 170 | .003517 | .066237 | .333416 | .59683 |
| 180 | 3.32226E-3 | 6.51552E-2 | .334047 | .597476 |
| 190 | 3.14795E-3 | 6.41355E-2 | .334622 | .598094 |
| 200 | 2.99103E-3 | 6.31717E-2 | .33515 | .598728 |

TABLE VII

Average weight of fragments weighing more than one grain from steel cylinders* filled with various cast explosives**

| Explosive | Composition (parts by weight) | Density (g/cc) | Detonation Velocity (m/sec) | \bar{M} (grains) |
|----------------|--|-------------------|--------------------------------|-----------------------|
| Baratol | 77 Barium Nitrate/23 TNT/0.1 Nitrocellulose | 2.49 | 4900 | 30 |
| Composition B | 59.5 RDX/39.5 TNT/1 Wax | 1.68 | 7900 | 12 |
| 75/25 Cyclotol | Any RDX-TNT combination, in this case 75 RDX/25 TNT | 1.69 | 8070 | 11.5 |
| H-6 | 47 RDX/31 TNT/22 Al/5 D-2 Wax | 1.73 | 7460 | 15.5 |
| HBX-1 | 40 RDX/38 TNT/17 Al/5 D-2 Wax | 1.71 | 7440 | 13.5 |
| HBX-3 | 31 RDX/29 TNT/35 Al/5 D-2 Wax | 1.81 | 7108 | 18 |
| Minol 2 | 40 Ammonium Nitrate/40 TNT/20 Al | 1.67 | | 19.5 |
| Pentolite | 50 PETN/50 TNT | 1.64 | 7530 | 13.5 |
| PTX-1 | 30 RDX/50 Tetrayl/20 TNT | 1.64 | 7730 | 12.5 |
| PTX-2 | 44 RDX/28 PETN/28 TNT 41 RDX/26 PETN/33 TNT | 1.67 | 7930 | 13 |
| Torpex | 45 RDX/37 TNT/18 Al | 1.80 | | 13 |
| Tritonal | 80 TNT/20 Al | 1.71 | | 17 |
| TNT | Trinitrotoluene | 1.58 | 6880 | 17.5 |

*9" long, 2" I.D., 2.5" O.D., AISI 1045 seamless, cold rolled, stress relief annealed, Rockwell hardness approximately 100-B.

**See Figure 8 for NOL data source; values of \bar{M} were assigned by fitting the data with a member of the one parameter family calculated with (6)-(8)

TABLE VIII

Average weight of fragments weighing more than one grain from steel cylinders* filled with various pressed explosives**

| Explosive | Composition (parts by weight) | Density (g/cc) | Detonation Velocity (m/sec) | \bar{M} (grains) |
|-----------------|--|-------------------|--------------------------------|-----------------------|
| Amatol | 80 Ammonium Nitrate/20 TNT | 1.64 | | 45 |
| BTWEN/Wax | 90 Bis trinitroethyl nitramine/10 Wax | 1.66 | 8180 | 10.5 |
| Composition A3 | 91 RDX/9 Wax | 1.61 | 8270 | 12 |
| Explosive D | Ammonium picrate | 1.53 | | 20 |
| MOX-2B | 54 Al/36 Ammonium Perchlorate/6 (97 RDX/3 Wax) | 1.99 | 4820 | 30 |
| Nitroguanidine | 4 TNT/2 Calcium Stearate/1 Graphite | 1.42 | | 20 |
| RDX/Wax (97/3) | 97 RDX/3 Wax | 1.61 | | 11.5 |
| RDX/Wax (95/5) | 95 RDX/5 Wax | 1.64 | 8450 | 11.5 |
| RDX/Wax (88/12) | 88 RDX/12 Wax | 1.59 | | 13 |
| RDX/Wax (85/15) | 85 RDX/15 Wax | 1.55 | 8330 | 15 |
| Tetryl | 2,4,6 Trinitrophenylmethylnitramine | 1.64 | 7460 | 17 |
| Pentolite | 50 PETN/50 TNT | 1.60 | 7400 | 16 |
| TNT | Trinitrotoluene | 1.54 | 6750 | 22 |

*9" long, 2" I.D., 2.5" O.D., AISI 1045 seamless, cold rolled, stress relief annealed, Rockwell harness approximately 100-B.

**See Figure 8 for NOL data source; values of \bar{M} were assigned by fitting the data with a member of the one parameter family calculated with (6)-(8)